MAX CAPM: Foundations for Low-Risk Anomalies, Mispricing, and Momentum*

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Abstract

We introduce an extension of the Capital Asset Pricing Model (CAPM) in which idiosyncratic lottery-like payoffs are priced in equilibrium. Lottery demand traders with a textbook probability weighting function overvalue stocks with high maximum returns, and interact with arbitrageurs who trade against mispricing. The model formalizes how extreme returns affect asset prices and provides a theory of the CAPM alpha. The model provides a micro-foundation and a common underlying explanation for the MAX effect, low-risk anomalies (the beta anomaly, the idiosyncratic volatility puzzle, and the overpricing of left-tail risk), aggregate mispricing, and momentum in stock returns. The model provides a foundation for constructing MAX-enhanced and MAX-weakened versions of the anomalies, the former which roughly double the baseline anomaly CAPM alphas and the latter which render the anomalies insignificant, consistent with the predictions of the model. We document that the anomalies are stronger in periods of higher aggregate lottery demand (even after controlling for sentiment) and that the results survive limits to arbitrage. Our findings show that probability weighting provides a unified theoretical and empirical explanation for six fundamental challenges to the CAPM and helps bridge the gap between rational and behavioral theories of asset prices.

JEL Classification: D8, G40, G41

Keywords: CAPM, Mispricing, MAX Effect, Momentum, Low-Risk Anomalies

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1 Introduction

The capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) was a major advance in financial theory. It remains widely used for mutual fund performance evaluation, capital budgeting and project valuation, and for tests of market efficiency and mispricing (Liu, Moskowitz and Stambaugh, 2021). Its theoretical development led to three Nobel prizes (to Markowitz, Tobin, and Sharpe), it provides the motivation for modern factor models in empirical finance, its parameter beta for each stock is widely reported to investors in financial media, and its equilibrium relation for the expected return on a stock is among the best-known equations in economics. Despite these developments, the CAPM has also faced persistent challenges from "anomalies" that earn positive abnormal returns (alphas) unexplained by the CAPM. Among the best known of these anomalies are the beta anomaly (Jensen, Black and Scholes, 1972; Frazzini and Pedersen, 2014) in which stocks with high market beta earn low abnormal returns, the idiosyncratic volatility puzzle (Ang et al., 2006) in which stocks with high idiosyncratic volatility earn low abnormal returns, momentum (Jegadeesh and Titman, 1993) in which stocks with low past returns (losers) earn low abnormal returns relative to stocks with high past returns (winners), the overpricing of stocks with high left tail risk (Atilgan et al., 2020; Kapadia et al., 2019; Wang, 2023) in which stocks with high tail risk earn low abnormal returns, the MAX effect (Bali, Cakici and Whitelaw, 2011) in which stocks with high maximum daily returns earn low abnormal returns, and the mispricing premium (Stambaugh and Yuan, 2017) in which stocks with low ex ante alphas earn low abnormal returns. These anomalies in particular have cast doubt on the empirical validity of the CAPM.

While behavioral economics has long recognized the role of behavioral investors in financial markets (Shiller, 2000; Akerlof and Shiller, 2010), more than a half-century after the introduction of the CAPM there is not yet a standard behavioral generalization of the CAPM that helps explain each of these six anomalies. This paper attempts to fill this gap. Here, we consider an extension of the CAPM based on one of the well-established biases in the behavioral economics literature: the tendency to overweight extreme payoffs driven by probability weighting (Wakker, 2010; Diecidue and Wakker, 2001). Models based on probability weighting have been increasingly applied to financial markets to explain an array of asset pricing phenomena at three different levels of aggregation (individual investors, individual stocks, and the aggregate market).[1] For the aggregate market, probability weighting has been applied to explain the equity premium puzzle (De Giorgi and Legg, 2012), the variance premium puzzle (Baele et al., 2019), the pricing kernel puzzle (Polkovnichenko and Zhao, 2013; Dierkes et al., 2023), and time variation in the market risk-return tradeoff (Ghazi, Schneider and Strauss, 2024). In the cross-section, probability weighting helps explain time variation in the slope of the security market line (Shi, Cui and Zhou, 2023). For individual investor behavior, probability weighting is linked to holding lottery-like stocks and under-diversified portfolios (Polkovnichenko, 2005; Dimmock et al., 2021).

In this paper, we derive an extension of the CAPM in which lottery demand traders deviate from mean-variance preferences by overweighting lottery-like payoffs via a standard textbook probability weighting function.[2] These traders interact in a market with arbitrageurs that have standard mean-variance preferences. In equilibrium, lottery demand traders are attracted to stocks with high maximum returns, while arbitrageurs are drawn to stocks with high CAPM alphas. The model provides a formal theory of the "CAPM alpha" that incorporates a role for idiosyncratic tail risk. The resulting "MAX CAPM" provides a micro-foundation for the MAX effect and its role in providing a unified theoretical explanation for properties of the six market anomalies noted above.

We empirically test the model predictions using the sample of US stocks over a period of 6 decades. A new theoretical prediction of the present model is that the anomalies will be strengthened by creating MAX-enhanced versions of the anomalies which invest in the subset of long-leg stocks with low maximum returns and short the subset of short-leg stocks with high maximum returns. The model also predicts that the anomalies will be weakened by creating MAX-weakened versions which invest in the subset of long-leg stocks with high maximum returns and short the subset of short-leg stocks with low maximum returns. We test this prediction for each of the anomalies and find that MAX-enhanced portfolios roughly double the anomaly CAPM alphas of the corresponding baseline anomalies, while MAX-weakened portfolios eliminate the anomalies.

[1] Empirical studies find that probability weighting also provides a unified explanation for behavior in lab experiments (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992), betting markets (Snowberg and Wolfers, 2010), insurance markets (Abito and Salant, 2019), state lottery markets (Lockwood et al., 2024), and contract markets (González-Jiménez, 2024).

[2] We use the non-extreme outcome expected utility (NEO-EU) weighting function which Wakker (2010) notes has a clearer and more convincing interpretation than other probability weighting functions. Further, Lockwood et al. (2024) find that it can explain their data on demand elasticities for state-run lotteries better than alternative weighting functions.

In particular, the baseline anomaly alphas range between 40 basis points and 71 basis points per month while the MAX-enhanced anomaly alphas range between 78 basis points and 147 basis points per month. Further, while the max-enhanced alphas are all significant at the 0.01 level, the max-weakened anomaly alphas range between -28 and 29 basis points per month and none are significant at the 0.05 level.[3] We find that these results survive limits to arbitrage.

A second theoretical prediction of the model is that the anomalies will be stronger in periods of higher aggregate lottery demand. Using a probability weighting-based measure of aggregate lottery demand from the recent literature (Ghazi, Schneider and Strauss, 2024; Ghazi et al., 2024), we find that each of the CAPM anomalies is stronger following periods of higher aggregate lottery demand, even after controlling for market sentiment.

Our paper makes three main contributions. First, we contribute to the literature on probability weighting in financial markets by introducing a new generalization of the CAPM that fuses the mean-variance framework with a textbook probability weighting function, while including heterogeneous investors (arbitrageurs and lottery demand traders). We use this framework to derive new theoretical predictions that we test empirically and for which we find empirical support. Second, we contribute to the literature on the MAX effect by providing it with a microfoundation and highlighting its role as providing a unified theoretical explanation for basic CAPM anomalies. Third, we contribute to the literature on six basic CAPM anomalies by showing theoretically and empirically that these anomalies are strengthened (weakened) by constructing MAX-enhanced (MAX-weakened) versions of the anomalies and that these results hold even when restricted to large cap stocks which are among the most liquid and easiest to arbitrage and that they hold in the modern era of trading technology (defined in Chen and Velikov (2023) as the post-2005 era), in which transaction costs have declined (Novy-Marx and Velikov, 2016), market efficiency has increased (Rösch, Subrahmanyam and Van Dijk, 2017), and many anomalies have disappeared (Chen and Velikov, 2023). Further, we show that a probability weighting-based measure of aggregate lottery demand predicts time variation in the CAPM alphas of the anomalies in the direction predicted by the theory. As a secondary contribution, we introduce an alternative measure of a stock's perceived

[3] Only the CAPM alpha for the MAX-weakened IVOL anomaly (-28 basis points per month) is marginally significant at the 0.10 level among the MAX-weakened portfolios. That reflects a positive premium for idiosyncratic risk among MAX-weakened portfolios. However, it is not a robust feature of our data as it disappears in the modern era of trading technology and is also eliminated using our alternative measure for a stock's maximum return.

maximum return and show that this measure yields similar empirical results, but is substantially less correlated with idiosyncratic volatility.[4] In this regard, the alternative MAX measure evades the critique of Hou and Loh (2016) that the MAX effect does not provide a valid explanation of the IVOL puzzle.

To elaborate on the first contribution we note that recent work applies probability weighting to the cross-section to extend the CAPM (Barberis, Jin and Wang, 2021; Driessen, Ebert and Koëter, 2021; Shi, Cui and Zhou, 2023) but has not yet delivered a structural formula for the CAPM alpha or provided the link between the CAPM alpha and the MAX effect that is central to our model. The prior work on extensions of the CAPM with probability weighting also does not consider different types of investors, instead using either representative agents who distort probabilities, or homogeneous investors that all have the same preferences. While prior work has combined prospect theory with the CAPM (Barberis and Huang, 2008; Barberis, Jin and Wang, 2021), the resulting model becomes too complex to obtain closed form results and the prior papers rely on simulations to study the model.[5] In contrast, in our setting the model can be solved in closed form even while accounting for the optimizing decisions of two types of investors (arbitrageurs and lottery demand traders). We show that the equilibrium exists and that it results in a closed form representation of the CAPM alpha. The model also generates the clear prediction that the anomalies can be strengthened or weakened by constructing MAX-enhanced or MAX-weakened versions, respectively, a prediction that does not emerge from the prior extensions of the CAPM and that stands in contrast to the vast theoretical literature that accentuates the left-tail but omits a preference for lottery demand.

Shi, Cui and Zhou (2023) obtain a three-moment CAPM with probability weighting in which co-skewness is priced. In contrast, our model involves the pricing of *idiosyncratic* risk. While Shi, Cui and Zhou (2023) apply their model to explain time variation in the beta anomaly, they do not consider the other anomalies that we study, nor do they construct max-enhanced and max-weakened versions of the beta anomaly. Driessen, Ebert and Koëter (2021) consider a representative agent

[4]The correlation between idiosyncratic volatility and maximum daily return is near 0.90, while the correlation between idiosyncratic volatility and the alternative *Max* measure is 0.64.

[5]As a consequence, the mechanism driving the anomalies is opaque and we only know that the anomalies are driven by the asset's degree of volatility, skewness, and capital gain overhang. Barberis, Jin and Wang (2021) consider a broad set of 23 anomalies which include the idiosyncratic volatility puzzle, momentum, and the MAX effect, though they do not consider the mispricing premium, the beta anomaly, or the MIN effect.

model in which the agent deviates from mean-variance preferences by using a similar probability weighting function to ours. They study a market with two assets, each with two possible outcomes, and derive their representation in terms of prices rather than as an extension of the familiar CAPM beta representation. They do not consider the anomalies that we consider, focusing instead on the implications of their setting for the pricing of an asset's skewness, volatility, and correlation. In contrast to the previous approaches, we provide a microfoundation for our model by considering both lottery demand investors and arbitrageurs with standard mean-variance preferences. This approach includes both rational and behavioral investors, thereby providing a more complete picture of the market. The theoretical predictions regarding max-enhanced and max-weakened versions of the anomalies and our predictions pertaining to the proportion of lottery demand traders in the market are new and particular to our setting.[6]

Regarding our second contribution, the proposed model formalizes how the ex ante maximum return on an asset as perceived by lottery demand traders affects investor behavior and prices. A distinctive feature of our model is that the MAX effect arises in equilibrium from the optimizing decisions of both types of agents and that the idiosyncratic MAX return on a stock is negatively related to the CAPM alpha. Our model contributes to the theoretical understanding of the MAX effect and predicts its unifying role in explaining properties of central CAPM anomalies.

Accounting for the role of the MAX effect as providing a unifying theme for the basic CAPM anomalies is a new theoretical development and provides testable predictions new to the literature. For instance, as noted by Jacobs, Regele and Weber (2015), none of the standard explanations for momentum implicate an important role for assets' maximum returns. In addition, while prior empirical work has linked the MAX effect to other anomalies, alternative theoretical models do not account for this. A standard theme in the modern asset pricing literature is to predict CAPM anomalies such as the IVOL puzzle and the overpricing of left tail risk in the opposite direction. In

[6]Our findings also contribute to the literature on extensions of the CAPM. Our approach differs from the mean-variance-skewness (MVS) CAPM (Kraus and Litzenberger, 1976; Harvey and Siddique, 2000; Schneider, Wagner and Zechner, 2020) in that under the MVS, only co-skewness is priced whereas in our approach both co-skewness (related to the market's maximum return) and idiosyncratic skewness (related to each asset's own idiosyncratic lottery-like payoff) are priced. Hong and Sraer (2016) consider a model with biased and unbiased investors in which a stock's own standard deviation is priced in equilibrium. They applied their model to explain the beta anomaly. In contrast, in our setting, a stock's own idiosyncratic extreme returns are priced in equilibrium and we study a broader set of anomalies. Theories of optimism bias have been developed (Ghirardato, Maccheroni and Marinacci, 2004; Bracha and Brown, 2012; Brunnermeier, Gollier and Parker, 2007) and applied to financial markets, but these models have not been developed into extensions of the CAPM as we do here.

particular, since models based on disaster risk (Barro, 2009; Wachter, 2013; Barro and Liao, 2021), disappointment aversion (Routledge and Zin, 2010; Schreindorfer, 2020), or ambiguity aversion (Gilboa and Schmeidler, 1989; Hansen and Sargent, 2001; Klibanoff, Marinacci and Mukerji, 2005) accentuate the left-tail, they predict stocks with high idiosyncratic volatility or high left-tail risk to earn positive abnormal returns. Yet this is in contrast to the empirical findings of Ang et al. (2006), Kapadia et al. (2019), Atilgan et al. (2020), and Wang (2023) that idiosyncratic risk and left-tail risk are negatively priced.

The proposed model helps to unify existing empirical findings regarding basic CAPM anomalies. Bali, Cakici and Whitelaw (2011) and Cheon and Lee (2018) provide evidence that the MAX effect explains the IVOL puzzle. Bali et al. (2017) show empirically that the MAX effect explains the beta anomaly. Kumar, Motahari and Taffler (2023) show empirically that the MAX effect helps explain the mispricing premium. Caglayan, Lawrence and Reyes-Peña (2023) show empirically that the MAX effect explains the overpricing of left-tail risk. However, the only precedent in empirical work for max-enhanced and max-weakened portfolios comes from Jacobs, Regele and Weber (2015) who constructed MAX-enhanced and MAX-weakened versions of momentum. They documented findings that support the predictions of our model. Jacobs, Regele and Weber (2015) conclude that their findings "appear to provide a challenge for popular theories of momentum, which are based on investor overreaction (Daniel, Hirshleifer and Subrahmanyam, 1998), investor underreaction followed by overreaction (Barberis, Shleifer and Vishny, 1998; Hong and Stein, 1999), agency issues in delegated fund management (Vayanos and Woolley, 2013), credit risk (Avramov et al., 2007) or the disposition effect (Grinblatt and Han, 2005)." They remark that their findings "do not fit neatly within a specific prominent theory of momentum." However, they did not have a theoretical model underlying their motivation for MAX-enhanced and MAX-weakened portfolios. We replicate the findings of Jacobs, Regele and Weber (2015) and demonstrate that they continue to hold in the modern era of trading technology, an era when the baseline momentum returns have disappeared. We also document similar findings for the other CAPM anomalies.

2 The MAX CAPM

We consider a market with two types of traders. Fraction $1-\varphi$ of traders are classical mean-variance traders who know the true conditional probability of P_{t+1}. We refer to them as *arbitrageurs*. The remaining fraction $\varphi \in (0,1)$ are biased traders who overweight the tails of the return distribution. We refer to them as *lottery demand traders* as they overweight lottery-like returns. The setting is a simple two-period market with n risky assets (stocks) and one risk-free asset (bond). We denote the periods with t and $t+1$. One share of stock j pays $p_{j,t+1}$ in the second period. Let $P_{t+1} := (p_{1,t+1}, \cdots, p_{n,t+1})'$, with the positive definite conditional covariance matrix $\Sigma_t := Var_t(P_{t+1})$. Risky assets are in fixed supply, and $\mathbf{1}_n := (1, 1, \cdots, 1)'$ denotes the normalized vector of outstanding shares. The bond has gross risk-free return R_f, which is set by the monetary authority that controls the supply of the bond.

Our approach extends the classical CAPM by considering a subset of traders who overweight lottery-like payoffs. In applications, the perceived maximum return could be the asset's historical maximum return such as the maximum daily return for an asset over the previous month, used by Bali, Cakici and Whitelaw (2011).

Our specification for lottery demand traders essentially merges mean-variance analysis with the NEO-EU model of probability weighting. Under the NEO-EU model (Chateauneuf, Eichberger and Grant, 2007), agents overweight the extreme payoffs and exhibit a preference toward assets with high maximum returns. Barberis and Huang (2008) links probability weighting to lottery demand in the stock market using simulations, while Lockwood et al. (2024) find that NEO-EU probability weighting best explains their findings on demand elasticicities for state-run lotteries. A novel feature of our approach is to provide a tractable extension of the CAPM which includes standard mean-variance agents who trade against mispricing (the arbitrageurs), and agents who deviate from mean-variance efficiency via a textbook probability weighting function (the lottery-demand traders).

As a simplifying assumption, we assume that the expectation, $E_t(R_{j,t+1})$, for the lottery demand traders (for a given asset j in period t) is the same as those of the arbitrageurs. This is a reasonable simplifying assumption since the lottery demand traders effectively truncate the return distribution at the asset's perceived extreme returns which are likely to be sufficiently far out in the tails that the truncation leaves the expected return virtually unchanged. Consequently, the preferences of the lottery demand traders and arbitrageurs only differ in that the former over-weight their perceived maximum and minimum returns.

Let x_j denote the holdings of asset j by the lottery demand traders, where we assume $x_j \geq 0$ for all j.[7] We consider a setting in which lottery demand traders determine their portfolio holdings according to the preferences in (1):

$$\max_{x} \sum_{j=1}^{n} x_j \, p_{j,t} \left((1-\gamma) E_t(R_{j,t+1}) + \gamma \Big(\theta \, \overline{R}_{j,t+1} + (1-\theta) \, \underline{R}_{j,t+1} \Big) - R_f \right) - \frac{\rho}{2} x' \Sigma_t x. \tag{1}$$

The perceived maximum return of stock j in period t is $\overline{R}_{j,t+1} := \frac{\overline{p}_{j,t+1}}{p_{j,t}}$, where $\overline{p}_{j,t+1}$ is the perceived ceiling price of the asset. The perceived minimum return of stock j is $\underline{R}_{j,t+1} := \frac{\underline{p}_{j,t+1}}{p_{j,t}}$, where $\underline{p}_{j,t+1}$ is the perceived floor price of the asset.

We denote the vector of perceived ceiling prices with $\overline{P}_{t+1} := (\overline{p}_{1,t+1}, \cdots, \overline{p}_{n,t+1})'$ and the vector of perceived floor prices with $\underline{P}_{t+1} := (\underline{p}_{1,t+1}, \cdots, \underline{p}_{n,t+1})'$. In (1), $\theta \in (0,1)$ reflects the degree of optimism of the lottery demand traders (the extent to which they overweight the lotterylike return), while $(1-\theta)$ reflects their degree of pessimism (the extent to which they overweight the likelihood of an extreme negative return such as a crash). Note that θ reflects the *direction* of the bias of the lottery demand traders, ranging from 0 (extreme pessimism) to 1 (extreme optimism). In comparison, $\gamma \in (0,1)$ reflects their *degree* of bias (the extent to which they deviate from textbook mean-variance preferences) ranging from 0 (no bias) to 1 (extreme sensitivity to tail events). Finally, $\rho > 0$ reflects the investors' risk aversion (which is the same for all agents).

The preferences in (1) essentially fuse mean-variance preferences at the heart of the classical CAPM with (a form of) textbook inverse-S shape probability weighting (Chateauneuf, Eichberger and Grant, 2007; Wakker, 2010).[8] Our specification of lottery demand traders essentially endows

[7]This assumption is consistent with the prohibitively large costs associated with short selling for individual investors. Kumar (2009) finds empirically that individual household investors have disproportionately large holdings of lotterylike stocks, whereas institutional investors have low holdings of such stocks. It thus seems reasonable to think of the lottery demand traders as representing individual household investors who generally avoid short selling. The assumption of non-negative holdings for a behavioral agent is also made in the representative agent asset pricing model in Zimper (2012).

[8]We make a simplifying assumption similar in spirit to that in Hong and Sraer (2016) in which we only focus on biased expectations (there is no bias in the variance-covariance matrix). Doing so simplifies the analysis and further clarifies the main insights of the model. Note also that the preferences in (1) can be viewed as agents who have correct expectations of both means and variances but also have a concern for robustness, represented by the classical Hurwicz criterion for robust optimization (Hurwicz, 1951). Under that interpretation, the agents in (1) care about

them with a NEO-EU probability weighting function of Chateauneuf, Eichberger and Grant (2007), which leads them to overweight the tails of the return distribution. NEO-EU agents exhibit a preference for lotterylike stocks with high maximum returns. This property is consistent with empirical properties of household portfolio choice such as the attraction for household investors to hold lottery-like stocks (Kumar, 2009; Dimmock et al., 2021).

One could alternatively interpret the setting as fusing the CAPM with a form of the α-maxmin model of choice under Knightian uncertainty (Ghirardato, Maccheroni and Marinacci, 2004), in which case γ represents the level of uncertainty. The model reduces to the CAPM when $\gamma = 0$ (there is no probability weighting or no uncertainty) or $\varphi = 0$ (there are no lottery demand traders).

Importantly, our specification of the lottery demand traders implies that idiosyncratic tail risk is priced in equilibrium. This property is consistent with the findings of Bégin, Dorion and Gauthier (2020) who find empirically that idiosyncratic Gaussian fluctuations in returns are easily diversifiable and are not priced, but that idiosyncratic tail risk is priced.

To complete the model setup, the mean-variance traders maximize (2).

$$\max_{y} \sum_{j=1}^{n} y_j \, p_{j,t} \left[E_t(R_{j,t+1}) - R_f\right] - \frac{\rho}{2} y' \Sigma_t y. \tag{2}$$

Let $x^* := (x_1, x_2, \cdots, x_n)'$ and $y^* := (y_1, y_2, \cdots, y_n)'$ denote the demand of the lottery demand traders and the arbitrageurs, respectively. In equilibrium, the market clears and (3) holds:

$$(1 - \varphi)x^* + \varphi \, y^* = \mathbf{1}_n. \tag{3}$$

The equilibrium of the model is defined as the set of prices $P_t = (p_{1,t}, \cdots, p_{n,t})'$, and allocations x^* and y^*, such that given the prices P_t, and the state variables $E_t P_{t+1}$, Σ_t, R_f, $\overline{P}_{t+1}$, $\underline{P}_{t+1}$, the allocations x^* and y^* solve the maximization problems of the lottery demand traders and mean-variance agents in (1) and (2), and the market clearing condition in (3) holds.

three features of asset returns: expected returns with respect to their prior distribution, risk or dispersion of returns with respect to their prior, and robustness of returns to a mis-specified prior, where the Hurwicz criterion is robust to all prior distributions over the same support.

2.1 Equilibrium Excess Returns

The equilibrium expected excess returns are given in the following proposition. In what follows, we drop the t subscripts for notational convenience in contexts where no confusion should arise.

PROPOSITION 1. *The market with a positive measure φ of lottery demand traders who maximize (1), a positive measure $(1-\varphi)$ of mean-variance traders who maximize (2), and the market clearing condition (3), has a unique equilibrium if the risk aversion parameter ρ is not too large, i.e., the inequality (10) in the Appendix is satisfied. The equilibrium expected excess return for asset j is the following:*

$$E(R_j) - R_f = \alpha_j + \beta_j\big(E(R_M) - R_f\big), \tag{4}$$

where $\beta_j := \frac{Cov(R_j,R_M)}{Var(R_M)}$, and the CAPM abnormal return α_j is given by:

$$\alpha_j := \left(\frac{\varphi\gamma}{1-\varphi\gamma}\right)\left(\theta(\beta_j\overline{R}_M - \overline{R}_j) + (1-\theta)(\beta_j\underline{R}_M - \underline{R}_j) + (1-\beta_j)R_f\right), \tag{5}$$

where, $R_M := \sum_{j=1}^{n} w_j R_j$ is the value-weighted market return with weights $w_j := \frac{p_j}{\sum_i p_i}$, and similarly, $\overline{R}_M := \sum_{j=1}^{n} w_j \overline{R}_j$, and $\underline{R}_M := \sum_{j=1}^{n} w_j \underline{R}_j$.

Proof. See the Online Appendix. □

Proposition 1 essentially provides a theory for the *CAPM alpha* (the asset-specific intercept in a time series regression of the assets' excess returns against the market excess return) which identifies variables that should affect an asset's CAPM abnormal returns. Under Proposition 1, the CAPM alpha for an asset, j, denoted α_j is amplified by the proportion of lottery demand traders, φ. Further, for stocks with higher maximum returns, the CAPM alpha is more negative.[9] More formally, the model predicts the following properties of CAPM abnormal returns:

COROLLARY 1. *(Properties of CAPM abnormal returns) Ceteris paribus, α_j is decreasing in the maximum return of stock j, $\overline{R}_j$. Moreover, this decline in α_j is larger if the fraction of lottery demand traders, φ, is high.*

[9] The model also makes predictions regarding the minimum return that are in line with the predictions of traditional asset pricing models based on disaster risk (Barro, 2009; Wachter, 2013; Barro and Liao, 2021), ambiguity aversion (Gilboa and Schmeidler, 1989; Hansen and Sargent, 2001; Klibanoff, Marinacci and Mukerji, 2005), and disappointment aversion (Routledge and Zin, 2010; Schreindorfer, 2020). The more novel theoretical prediction from the present model's extension of the CAPM pertains to the right tail. Consequently, we focus on those predictions.

Corollary 1 provides a formal connection between the MAX effect and the CAPM alpha (abnormal returns), demonstrating that the MAX effect directly leads to overpricing. Consequently, stocks with sufficiently high $\overline{R}_j$ are overpriced relative to the CAPM (they have $\alpha_j < 0$).

Since the model given by (4) and (5) generalizes the classical CAPM to incorporate the pricing of a stock's maximum return, we refer to it as the *MAX CAPM*.

2.2 Implications for Investor Behavior

In the model, lottery demand investors are attracted toward lotterylike stocks (assets with high maximum returns), which leads them to under-diversify to enjoy a small chance of a large payoff. These predictions are supported empirically by Dimmock et al. (2021) who measure the degree of probability weighting for household investors and document that "higher probability weighting is associated with owning lottery-type stocks," and that people with higher probability weighting tend to under-diversify.

Here we briefly consider the model implications for investor behavior for assets whose perceived ceiling and floor prices have not changed from the previous period.

COROLLARY 2. *For assets whose maximum and minimum prices have not changed:*

1. *(Trading of assets following an increase in their prices): As the price of an asset increases toward the perceived ceiling price of the lottery demand traders, the lottery demand traders lower their holdings, and arbitrageurs increase their holdings. In contrast, as the price decreases toward the perceived floor price, lottery demand traders increase their holdings, and arbitrageurs decrease their holdings.*

2. *(Trading of assets following an increase in their alphas): Assuming an individual asset has a negligible effect on the market, ceteris paribus, as the price of an asset increases toward the perceived ceiling price, the asset's perceived maximum return decreases, the asset's alpha increases, and lottery demand traders sell the asset whereas arbitrageurs buy it.*

Proof. See the Online Appendix. □

Corollary 2 predicts that biased traders will increasingly sell stocks as their prices increase toward their perceived ceiling price and will increasingly buy stocks as their prices decrease far below the ceiling price. The corollary further predicts that arbitrageurs will do the opposite. In particular, under Part 2 of the corollary, arbitrageurs trade against mispricing, buying stocks whose CAPM alphas have increased, whereas lottery demand traders sell those stocks and buy lottery-like stocks whose prices have decreased far below their perceived ceiling price. Under the model, lottery demand traders perceive such stocks to have an increased lottery-like return.

Two questions to address in models with behavioral investors are why mispricing is not corrected and what motivates behavioral investors to trade in the first place, especially if they are on the losing side of the trades. Regarding the first question, since in the model the arbitrageurs are risk-averse, their demands are finite so that mispricing is not fully corrected. This is a standard approach to limiting mispricing correction as noted by Cochrane (2011).[10]

Regarding the second question, note that both lottery demand traders and arbitrageurs have plausible motives for trading as they each perceive themselves as "buying low" and "selling high". Arbitrageurs buy underpriced stocks and sell overpriced stocks, whereas lottery demand traders buy stocks with low prices relative to their perceived ceiling price and sell stocks with high prices relative to their perceived ceiling price.

3 Model Implications for CAPM Anomalies

We focus on six prominent CAPM anomalies in which attraction to lottery-like payoffs provides a plausible intuitive explanation. These anomalies include (i) the MAX effect (Bali, Cakici and Whitelaw, 2011) in which stocks with high maximum returns earn low abnormal returns; (ii) the mispricing premium (Stambaugh and Yuan, 2017) in which stocks with high ex ante abnormal returns earn high abnormal returns; (iii) momentum (Jegadeesh and Titman, 1993) in which stocks with high (low) past returns earn high (low) abnormal returns; and three low-risk anomalies: (iv) the low abnormal returns to stocks with high market beta (the beta anomaly) (Jensen, Black and Scholes, 1972); (v) the low abnormal returns to stocks with high idiosyncratic volatility (the idiosyncratic volatility (IVOL) puzzle) (Ang et al., 2006); and (vi) the low abnormal returns to stocks with high left-tail risk (Atilgan et al., 2020). The low returns to stocks with high systematic

[10] As Cochrane (2011) remarks, "There are some frictions in many behavioral models, but these are largely secondary and defensive, to keep risk-neutral "rational arbitrageurs" from coming in and undoing the behavioral biases. Often, simple risk aversion by the rational arbitrageurs would serve as well."

risk, high idiosyncratic risk, or high left-tail risk could plausibly be due to these assets also having high maximum returns. Similarly, the mispricing premium could be due to overpriced stocks with high maximum returns. For momentum, loser stocks plausibly have low prices relative to their ceiling price due to their low past returns, in which case the MAX CAPM predicts that they have high maximum returns.

The set of six anomalies that we consider is also an important set as it contains many of the seminal empirical challenges to the predictions of equilibrium asset pricing theory. For instance, the beta anomaly challenges the prediction of the CAPM that the security market line is upward sloping. The mispricing premium challenges the prediction of the CAPM that the CAPM alpha is not significantly different from zero. The idiosyncratic volatility puzzle challenges the prediction of the CAPM that idiosyncratic risk is not priced, and it challenges the prediction of the Merton (1987) model that idiosyncratic risk is positively priced. The overpricing of left-tail risk challenges the prediction of models based on disaster risk (Barro, 2009; Wachter, 2013; Barro and Liao, 2021), ambiguity aversion (Gilboa and Schmeidler, 1989; Hansen and Sargent, 2001; Klibanoff, Marinacci and Mukerji, 2005), and disappointment aversion (Routledge and Zin, 2010; Schreindorfer, 2020) that left-tail risk is positively priced. Momentum challenges the prediction of the efficient market hypothesis that future abnormal returns are not predictable from past returns. Despite the very different theoretical predictions these fundamental anomalies challenge, we use the present model to investigate whether they can be given a unified explanation and a microfoundation.

The following corollary implicates the difference in maximum returns between the short and long legs of the anomalies as a driver of each anomaly. In this respect, the model predicts that the MAX effect of Bali, Cakici and Whitelaw (2011) provides a unifying role in helping to explain these basic CAPM anomalies.

COROLLARY 3. *(Implications for CAPM Anomalies): Consider a long-short portfolio that takes a long position in asset L and a short position in asset S. Under Equations (4) and (5), the CAPM alpha, α_{LS}, for this portfolio is increasing in $\frac{\varphi\gamma}{(1-\varphi\gamma)}\theta(\overline{R}_S - \overline{R}_L)$. Further, $\alpha_{LS} > 0$ if $\theta(\overline{R}_S - \overline{R}_L)$ is sufficiently large (i.e., if inequality (6) holds).*

$$\theta(\overline{R}_S - \overline{R}_L) > (\beta_S - \beta_L)(\theta\overline{R}_M + (1-\theta)\underline{R}_M - R_f) + (1-\theta)(\underline{R}_L - \underline{R}_S). \tag{6}$$

For the CAPM anomalies that we study, this implies:

1. *(MAX Effect) If the long (short) leg asset has low maximum returns, LM (high maximum returns, HM), the CAPM alpha,* α_{MAX}*, is increasing in* $\frac{\varphi\gamma}{(1-\varphi\gamma)}\theta(\overline{R}_{HM} - \overline{R}_{LM})$.

2. *(Overpricing of left-tail risk) If the long (short) leg asset has low left-tail risk, LTR (high left-tail risk, HTR), the CAPM alpha,* α_{MIN}*, is increasing in* $\frac{\varphi\gamma}{(1-\varphi\gamma)}\theta(\overline{R}_{HTR} - \overline{R}_{LTR})$.

3. *(IVOL Premium) If the long (short) leg asset has low idiosyncratic volatility, LV (high idiosyncratic volatility, HV), the CAPM alpha,* α_{IVOL}*, is increasing in* $\frac{\varphi\gamma}{(1-\varphi\gamma)}\theta(\overline{R}_{HV} - \overline{R}_{LV})$.

4. *(Betting-against-Beta Premium) If the long (short) leg asset has low market beta, LB (high market beta, HB), the CAPM alpha,* α_{BAB}*, is increasing in* $\frac{\varphi\gamma}{(1-\varphi\gamma)}\theta(\overline{R}_{HB} - \overline{R}_{LB})$.

5. *(Mispricing Premium) If the long (short) leg is an underpriced asset, U, (overpriced asset, O), for which* $\alpha_U > 0 > \alpha_O$*, the CAPM alpha,* α_{UMO}*, is increasing in* $\frac{\varphi\gamma}{(1-\varphi\gamma)}\theta(\overline{R}_{O} - \overline{R}_{U})$.

6. *(Momentum) If the long (short) leg is a winner asset, W, with a high past return (a loser asset, L, with a low past return), the CAPM alpha,* α_{WML}*, is increasing in* $\frac{\varphi\gamma}{(1-\varphi\gamma)}\theta(\overline{R}_{L} - \overline{R}_{W})$.

We use the model predictions from Corollary 3 to generate precise testable hypotheses. Note that for each anomaly, the corollary highlights a role for the difference in maximum returns between the long and the short leg (a cross-sectional component of the anomalies), and a role for the proportion of lottery demand traders in the market (a systematic component of the anomalies that may vary across time). To evaluate the cross-sectional component of the anomalies predicted by Corollary 3, we form *max-enhanced* and *max-weakened* portfolios. Consider a generic long-short portfolio. A max-enhanced portfolio takes a long position in the subset of long-leg stocks with low maximum returns and a short position in the subset of short-leg stocks with high maximum returns. A max-enhanced portfolio is designed to expand the difference in maximum returns ($\overline{R}_{HTR} - \overline{R}_{LTR}$, $\overline{R}_{HV} - \overline{R}_{LV}$, $\overline{R}_{HB} - \overline{R}_{LB}$, $\overline{R}_{O} - \overline{R}_{U}$, and $\overline{R}_{L} - \overline{R}_{W}$). Under the MAX CAPM, a max-enhanced portfolio should strengthen each of these anomalies.

A max-weakened portfolio takes a long position in the subset of long-leg stocks with high maximum returns and a short position in the subset of short-leg stocks with low maximum returns. A max-weakened portfolio is designed to shrink the difference in maximum returns between the long and short leg assets. Under the MAX CAPM, a max-weakened portfolio should weaken each of these anomalies.

Our first three hypotheses pertain to the the cross-sectional variation in anomalies.

HYPOTHESIS 1. *A max-enhanced version of the MIN, IVOL, BAB, UMO, and WML anomalies yields CAPM abnormal returns, $\alpha_{\overline{MIN}}$, $\alpha_{\overline{IVOL}}$, $\alpha_{\overline{BAB}}$, $\alpha_{\overline{UMO}}$, and $\alpha_{\overline{WML}}$, that are positive and significant.*

If the anomalies are driven mainly by the difference in maximum returns, then a max-weakened version of the anomalies could render the anomalies insignificant.

HYPOTHESIS 2. *A max-weakened version of the MIN, IVOL, BAB, UMO, and WML anomalies yields CAPM abnormal returns, $\alpha_{\underline{MIN}}$, $\alpha_{\underline{IVOL}}$, $\alpha_{\underline{BAB}}$, $\alpha_{\underline{UMO}}$, and $\alpha_{\underline{WML}}$, that are not significant.*

HYPOTHESIS 3. *The max-enhanced MIN, IVOL, BAB, UMO, and WML anomalies yield CAPM abnormal returns, $\alpha_{\overline{MIN}}$, $\alpha_{\overline{IVOL}}$, $\alpha_{\overline{BAB}}$, $\alpha_{\overline{UMO}}$, and $\alpha_{\overline{WML}}$, that are significantly larger than those of the corresponding max-weakened anomalies, $\alpha_{\underline{MIN}}$, $\alpha_{\underline{IVOL}}$, $\alpha_{\underline{BAB}}$, $\alpha_{\underline{UMO}}$, and $\alpha_{\underline{WML}}$.*

Our fourth hypothesis pertains to the time series variation in the anomalies.

HYPOTHESIS 4. *The max effect and the max-enhanced versions of the anomalies have higher abnormal returns following periods in which aggregate investor lottery demand is higher.*

To evaluate our fourth hypothesis, we use a theory-based index of time variation in lottery demand from Ghazi et al. (2024a). The index is given by $-\sigma_t(1-\theta_t)$ where σ_t is a measure of the conditional market volatility, estimated from a GARCH model, and θ_t is the measure of market optimism from Ghazi et al. (2024b) that is estimated from an asset pricing model with a representative NEO-EU agent. Ghazi et al. (2024a) motivate the index by showing analytically that the marginal value from participating in the stock market for a NEO-EU investor is increasing in market optimism, θ, and decreasing in market volatility, σ. A high index value indicates a period of high investor lottery demand in which lottery demand investors who overweight assets with high maximum returns have their largest influence on the market. A low index value indicates a period of low investor lottery demand in which lottery demand investors withdraw from the market. In light of this observation, one could view the index $-\sigma_t(1-\theta_t)$ as reflecting time variation in φ (the proportion of lottery demand traders in the market).[11]

Corollary 3 focuses on a component of anomalies predicted by the MAX CAPM (assets' maximum returns) that is absent from traditional asset pricing models. For instance, under the CAPM, neither the left or right tail is overweighted (beyond standard risk aversion) and in models based on ambiguity (Gilboa and Schmeidler, 1989; Hansen and Sargent, 2001; Klibanoff, Marinacci and Mukerji, 2005), disappointment aversion Routledge and Zin (2010); Schreindorfer (2020), or disaster risk Barro (2009); Wachter (2013); Barro and Liao (2021), only the left tail is overweighted.

Corollary 3 helps unify a variety of existing empirical findings. Part 1 of the corollary (the MAX effect) is supported empirically by Bali, Cakici and Whitelaw (2011), and Cheon and Lee (2018). Part 2 of the corollary is supported empirically by Caglayan, Lawrence and Reyes-Peña (2023) who find that the MAX effect explains the low returns to stocks with low minimum returns. Part 3 of the corollary is supported empirically by Bali, Cakici and Whitelaw (2011) who finds that the MAX effect explains the idiosyncratic volatility puzzle.[12] Part 4 of the corollary is supported empirically by Bali et al. (2017) who find that the MAX effect explains the beta anomaly. Part 5 of the corollary is supported empirically by Kumar, Motahari and Taffler (2023) who find that the MAX effect helps explain the mispricing premium. Part 6 of the corollary is supported empirically by Jacobs, Regele and Weber (2015) who find that the MAX effect is an important driver of momentum. One contribution of the present paper is to provide a theory which unifies these empirical findings by linking the MAX effect to these anomalies.

While prior work has investigated the role of the MAX effect in isolated anomalies, we test the model predictions using a common stock universe (from CRSP) for all six anomalies. We conduct our tests excluding microcap stocks as recommended by Hou, Xue and Zhang (2020). We show in our robustness analyses that results are similar using all stocks. We show that our results

[11]Note that θ_t is also expected to drive variation in the weight on the maximum return, θ, relative to the minimum return, $1-\theta$. As higher θ_t is predicted to increase both φ and θ, its predicted effect on CAPM alphas is unambiguous.

[12]Subsequent work by Hou and Loh (2016) has dismissed the MAX effect as an explanation for the IVOL puzzle since, although the MAX effect can explain the puzzle, the cross-sectional correlation between a stock's maximum daily return and a stock's idiosyncratic volatility is very high (approximately 0.90). Hou and Loh (2016) argues that because of this high correlation, the MAX effect does not provide an adequate explanation for the IVOL puzzle. In our robustness analyses, we use an alternative method for calculating an asset's maximum return (analogous to how Atilgan et al. (2020) compute value-at-risk, but applied to the right tail). We show that this alternative measure also supports the model predictions and it provides perhaps a more satisfactory explanation for the IVOL puzzle since its cross-sectional correlation with idiosyncratic volatility is substantially lower at 0.64.

survive limits to arbitrage by restricting the analyses to large cap stocks and to the modern era of trading technology, starting in 2006 (Chen and Velikov, 2023), in which transaction costs have declined (Novy-Marx and Velikov, 2016), market efficiency has increased, (Rösch, Subrahmanyam and Van Dijk, 2017), and many anomalies have disappeared (Chen and Velikov, 2023).

4 Data and Methodology

We obtain data on ordinary common stocks traded on NYSE, AMEX, and NASDAQ from CRSP for the period of July 1963 to December 2023. We calculate the following characteristics. *Ivol* is the idiosyncratic volatility calculated as the standard deviation of residuals from estimating a regression of daily returns on Fama-French three factors over a month. We calculate *Beta* using five years of monthly returns with at least 36 observations (hence, *Beta* starts in July-1968). *Max* is the maximum daily return during a month. *Max5*, our alternative measure of maximum return is the 95^{th} percentile of daily returns over the past year. We measure left tail risk, *Min*, by calculating the 5% value-at-risk times negative one over the past year. *Momentum* is the cumulative monthly return over the past 11 months ending one month prior to each month. We obtain mispricing measures (*Misp*, from July-1965 to December-2016) for individual stocks from Robert Stambaugh's website. Data on market return and risk-free rate is from Prof. Kenneth French's webiste.

We construct a baseline portfolio by sorting the stocks in each month into an anomaly quintiles. The baseline portfolio is a long-short portfolio of stocks in the extreme quintiles that generate a positive abnormal return according to the literature. To construct enhanced and weakened portfolios, we first sort stocks into *Max* quintiles. Within each quintile, we sort the stocks into quintiles for each anomaly. For the long-leg (short-leg), we buy (sell) stocks in the relevant extreme anomaly quintile within the bottom (top) *Max* quintile. For instance, the *Ivol*-enhanced portfolio buys (sells) stocks in the bottom (top) *Ivol* quintile within the bottom (top) *Max* quintile. We mitigate the effect of small cap stocks by performing all sorting exercises based on NYSE breakpoints. Furthermore, all portfolios are value-weighted. In addition, we exclude micro cap stocks in our main analyses. Micro cap stocks have a market capitalization below the 20^{th} percentile based on NYSE breakpoints.

To evaluate portfolio performance and the predictions of the model, we run a time-series regression of a long-short portfolio returns on the market excess return. We report the intercept of the estimated regression (α) and the coefficent of the market excess return (β). Further, we regress the residuals on the lagged lottery demand and report the estimated coefficient to evaluate the predictability of portfolio returns with aggregate lottery demand.

4.1 Summary Statistics

Table 1, Panel A, provides basic summary statistics for the sorting variables used in our analysis to form the portfolios including the number of stocks, their mean, and standard deviation, and their first, second, and third quartiles. Panel B displays the cross-sectional correlations between the sorting variables and is consistent with prior findings, for instance that *Max* has a high cross-sectional correlation with *Ivol* of 0.86 (Bali, Cakici and Whitelaw, 2011; Hou and Loh, 2016).

Table 1. Summary Statistics

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A	N	Mean	STD	25%	50%	75%
Return	1178997	1.8%	13.1%	−4.6%	1.1%	7.2%
Ivol	1178962	1.8%	1.3%	1.0%	1.5%	2.2%
Beta	944548	1.12	0.60	0.73	1.06	1.43
Momentum	1117643	0.21	0.69	−0.08	0.12	0.36
Misp	973852	49.14	13.04	39.86	48.47	57.82
Min	1157652	3.5%	1.7%	2.3%	3.1%	4.2%
Max	1178996	5.1%	4.8%	2.6%	3.9%	6.2%
Max5	1157652	3.9%	2.1%	2.5%	3.4%	4.7%

Panel B	*Return*	*Ivol*	*Beta*	*Momentum*	*Misp*	*Min*	*Max*	*Max5*
Return	1							
Ivol	0.19	1						
Beta	0.03	0.26	1					
Momentum	0.00	0.09	0.08	1				
Misp	−0.01	0.16	0.11	−0.15	1			
Min	0.05	0.63	0.43	0.06	0.22	1		
Max	0.39	0.86	0.24	0.06	0.13	0.52	1	
Max5	0.12	0.64	0.43	0.25	0.18	0.93	0.53	1

This table reports the summary statistics (Panel A) and correlations (Panel B) of the stock level variables. *Return* is the monthly stock return. *Ivol* is the idiosyncratic volatility calculated as the standard deviation of residuals from estimating a regression of daily returns on Fama-French three factors over a month. *Beta* is calculated using five years of monthly returns with at least 36 observations. *Max* is the maximum daily return during a month. *Max5*, our alternative measure of maximum return is the 95^{th} percentile of daily returns over the past year. *Min* is the 5^{th} percentile of daily returns over the past year times negative one. *Momentum* is the cumulative monthly return over the past 11 months ending one month prior to each month. *Misp* is the mispricing measure from Robert Stambaugh's website. The sample period is July 1963 to December 2023 (July-1968 to December-2023 for Beta, June-1965 to December-2016 for *Misp*) and excludes micro cap stocks defined as stocks in the bottom 20^{th} percentile of market capitalization each month using NYSE breakpoints.

Table 2 displays summary statistics of the baseline, enhanced, and weakened portfolios used in our analysis. On a nominal basis, all enhanced portfolios produce higher average returns than their corresponding baseline and weakened portfolios. We observe generally consistent results using the Sharpe ratio. In the next section, we compare the performance of these portfolios using the CAPM model and find empirical support for the model predictions from Section 2.

Table 2. Descriptive Statistics of Anomaly Returns

Baseline	Mean	Min	Max	STD	SR
IVOL	0.09%	−22.16%	22.87%	4.74%	0.07
BAB	0.02%	−25.97%	21.55%	5.73%	0.01
WML	0.58%	−35.36%	22.58%	5.37%	0.37
UMO	0.54%	−12.99%	15.67%	3.23%	0.58
MIN	−0.03%	−30.44%	25.65%	6.39%	−0.02
MAX	0.04%	−21.89%	24.16%	4.72%	0.03
Enhanced	Mean	Min	Max	STD	SR
IVOL	0.38%	−36.49%	31.33%	7.44%	0.18
BAB	0.06%	−47.86%	28.26%	8.89%	0.02
WML	1.00%	−45.25%	30.46%	7.67%	0.45
UMO	0.78%	−25.25%	32.48%	6.66%	0.40
MIN	0.21%	−48.87%	35.23%	9.57%	0.08
Weakened	Mean	Min	Max	STD	SR
IVOL	−0.09%	−17.99%	16.40%	4.29%	−0.07
BAB	−0.05%	−29.87%	16.01%	3.98%	−0.05
WML	0.44%	−35.60%	38.90%	6.81%	0.23
UMO	0.50%	−23.03%	27.24%	4.83%	0.36
MIN	−0.18%	−15.32%	13.36%	3.48%	−0.18

This table reports the descriptive statistics of the baseline, enhanced, and weakened portfolios. STD is the standard deviation of returns. SR is the annualized Sharpe ratio, calculated as the average returns divided by the standard deviation of returns for each long-short portfolio times the square root of 12. Baseline portfolios are constructed by buying (selling) stocks in the top quintile (bottom) of each anomaly. For momentum (*WML* portfolios), the trade is reverse. For the enhanced portfolios, the long-leg (short-leg) consists of the stocks in the bottom quintile of each anomaly within the bottom (top) quintile of maximum return (*Max*). For the weakened portfolios, the long-leg (short-leg) consists of the stocks in the bottom quintile of each anomaly within the top (bottom) quintile of maximum return (*Max*). The corresponding characteristics for the portfolios are from Table 1 and are as follows: idiosyncratic volatility for *IVOL*, beta for *BAB* (bettign against beta), momentum for *WML* (winners minus losers), mispricing for *UMO* (underpriced minus overpriced), minimum return for *MIN*. The sample period is July 1963 to December 2023 (July-1968 to December-2023 for *BAB*, June-1965 to January-2017 for *UMO*) and excludes micro cap stocks defined as stocks in the bottom 20^{th} percentile of market capitalization each month using NYSE breakpoints. NYSE breakpoints are used to sort stocks into quintiles of characteristics. All portfolios are value weighted.

5 Cross-Sectional Variation in Anomalies due to MAX Returns

In this section, we examine the theoretical predictions of the model in the cross-section of stock returns. In particular, we construct Max-enhanced and Max-weakened portfolios of the anomalies and test the model predictions in Hypotheses 1 through 3. We then examine the effects of limits to arbitrage.

5.1 Performance of Baseline Portfolios

We start by constructing baseline portfolios for the six anomalies and examine their performance relative to the CAPM. Panel A of Table 3 reports the CAPM alpha and beta of each portfolio. Consistent with the literature, all six anomaly alphas are significant at least at the 0.05 level and range from monthly values of 0.40% (4.8% annualized) for the MAX effect portfolio to 0.71% (8.52% annualized) for the mispricing premium (UMO).

5.2 Performance of MAX-Enhanced and MAX-Weakened Portfolios

Table 3 also summarizes the performance of the max-enhanced and max-weakened portfolios as well as their relative performance for each anomaly. Panel B of Table 3 shows that the monthly CAPM alphas for the max-enhanced portfolios range from 0.78% (9.36%) annualized for the max-enhanced BAB portfolio to 1.47% (17.64% annualized) for the max-enhanced momentum portfolio. These results support Hypothesis 1, documenting that the max-enhanced version of the MIN, IVOL, BAB, UMO, and WML anomalies yield large positive and significant CAPM abnormal returns.

Panel C of Table 3 shows that the monthly CAPM alphas for the max-weakened portfolios range from -0.28% (-3.36%) annualized for the max-weakened IVOL portfolio to 0.29% (3.48% annualized) for the max-weakened UMO portfolio. None of the alphas for the max-weakened portfolios are significant at the 0.05 level. Only the IVOL premium which becomes negative (reflecting a positive premium for idiosyncratic risk) is marginally significant at the 0.10 level. These results support Hypothesis 2, documenting that the max-weakened versions of the MIN, IVOL, BAB, UMO, and WML anomalies yield insignificant CAPM alphas.

Panel D of Table 3 compares the max-enhanced and max-weakened portfolios for each anomaly. The table reveals that in each case, the max-enhanced portfolios earn significantly higher CAPM

Table 3. Performance of Baseline, Enhanced, and Weakened Strategies

Panel A	(1)	(2)	(3)	(4)	(5)	(6)
Baseline	IVOL	BAB	WML	UMO	MIN	MAX
α	0.45^{***}	0.49^{***}	0.68^{***}	0.71^{***}	0.51^{**}	0.4^{**}
	(2.61)	(2.70)	(3.45)	(5.92)	(2.48)	(2.36)
β	-0.63^{***}	-0.84^{***}	-0.17^{*}	-0.34^{***}	-0.95^{***}	-0.64^{***}
	(−10.75)	(−12.22)	(−1.79)	(−5.84)	(−13.99)	(−9.59)
N	725	666	725	618	725	725
Adj. R-squared	0.35	0.46	0.02	0.22	0.44	0.37
Panel B						
Enhanced	IVOL	BAB	WML	UMO	MIN	
α	0.9^{***}	0.78^{***}	1.47^{***}	1.23^{***}	1.0^{***}	
	(3.47)	(2.80)	(5.93)	(4.41)	(3.35)	
β	-0.92^{***}	-1.3^{***}	-0.82^{***}	-0.89^{***}	-1.39^{***}	
	(−8.82)	(−12.91)	(−6.97)	(−9.13)	(−13.80)	
N	725	666	722	618	725	
Adj. R-squared	0.31	0.45	0.23	0.36	0.43	
Panel C						
Weakened	IVOL	BAB	WML	UMO	MIN	
α	-0.28^{*}	−0.04	0.1	0.29	−0.15	
	(−1.92)	(−0.21)	(0.35)	(1.57)	(−1.23)	
β	0.34^{***}	−0.03	0.61^{***}	0.4^{***}	−0.05	
	(5.61)	(−0.72)	(7.37)	(5.98)	(−1.53)	
N	725	666	725	618	725	
Adj. R-squared	0.13	0.00	0.16	0.14	0.00	
Panel D						
Enhanced - Weakened	IVOL	BAB	WML	UMO	MIN	
α	1.19^{***}	0.82^{***}	1.39^{***}	0.93^{**}	1.16^{***}	
	(3.41)	(2.68)	(3.49)	(2.35)	(3.69)	

This table reports the performance of the baseline (Panel A), enhanced (Panel B) and weakened (Panel C) portfolios in Table 2. *IVOL* represents idiosyncratic volatility anomaly. *BAB* represents betting against beta. *WML* represents momentum. *UMO* represents the mispricing of Stambaugh and Yuan (2017). *MIN* represents the left-tail risk. *MAX* represents maximum return. α, in percent, is the intercept of regressing portfolio return on market excess return. β is the coefficient on the market excess return. The sample period is July 1963 to December 2023 (July-1968 to December-2023 for *BAB*, June-1965 to January-2017 for *UMO*). The test statistics, in parentheses, are based on Newey-West standard errors with 12 lags. ***,**, and * represent statistical significance at 1%, 5%, and 10%, respectively.

alphas than the max-weakened portfolios with the difference ranging between monthly alphas of 0.82% (9.84% annualized) for BAB to 1.39% (16.68% annualized) for momentum. These results support Hypothesis 3, documenting that the max-enhanced MIN, IVOL, BAB, UMO, and WML anomalies yield CAPM abnormal returns that are significantly larger than those of the corresponding max-weakened anomalies. Our findings reveal that probability weighting provides a mechanism by which the standard CAPM anomalies are strengthened (through max-ehnanced portfolios) and by which they are eliminated (through max-weakened portfolios). The MAX CAPM can thus help to simultaneously reconcile these systematic deviations from the CAPM.

5.3 Limits to Arbitrage: Performance restricted to Large Cap Stocks

We next consider if the results in the previous subsection are robust to two basic limits to arbitrage. First, we construct the anomalies restricted to large cap stocks (those with above-median NYSE market capitalization) which are among the most liquid and easiest to arbitrage. If the performance of the baseline and enhanced portfolios is driven by limits to arbitrage, one might expect the CAPM alphas to be eliminated for large cap stocks. In the next subsection, we restrict the analyses to the modern era of trading technology (which Chen and Velikov (2023) specify as the era beginning 2006:01). In this recent period, transaction costs have declined (Novy-Marx and Velikov, 2016), market efficiency has increased (Rösch, Subrahmanyam and Van Dijk, 2017), and many anomalies have disappeared (Chen and Velikov, 2023).

Table 4 summarizes the performance of the baseline, max-enhanced, and max-weakened portfolios using all available data for the anomalies restricted to large cap stocks. Panel A reveals that all six anomalies exist among large cap stocks, earning significant CAPM alphas that range from 35 basis points per month (for IVOL) to 63 basis points per month (for UMO).

Panel B of Table 4 reveals that the max-enhanced versions of BAB, IVOL MIN, UMO, and WML each earn significant CAPM alphas ranging from 62 basis points per month (for BAB) to 112 basis points per month for WML. This finding further supports Hypothesis 1 that the max-enhanced portfolios earn positive and significant CAPM alphas.

Panel C of Table 4 documents that the max-weakened versions of BAB, WML, UMO, and MIN are not significant at the 0.05 level for large cap stocks, while IVOL is significant in the opposite direction (i.e., the premium for idiosyncratic risk is positive) for large cap stocks.[13]

Panel D of Table 4 shows that the max-enhanced portfolios generally outperform the max-weakened portfolios for large cap stocks. The out performance is economically significant in all cases (ranging from a difference in alphas of 7.2% (annualized) for UMO to 13.2% (annualized) for IVOL. The differences are mostly statistically significant.

5.4 Limits to Arbitrage: Performance in the Modern Era of Trading Technology

Following advances in decimalization and online trading in the early 2000's, trading costs and associated barriers to arbitrage have fallen. Chen and Velikov (2023) define the modern era of trading technology as the period beginning January, 2006. They show that many anomalies have disappeared in the modern era. In this section, we investigate whether our benchmark results continue to hold when restricting our analysis to the modern era of trading technology.

Table 5 shows the performance of the baseline, enhanced, and weakened strategies in the period starting 2006:01. Panel A displays the results for the baseline strategies. While the baseline BAB and momentum strategies have largely disappeared in the modern era (earning insignificant alphas of 35 basis points per month), the other four baseline strategies earn significant alphas ranging from 53 basis points per month (4.2% annualized) for IVOL to 72 basis points per month (8.64% annualized) for MAX and UMO.

Panel B of Table 5 documents that all enhanced versions of the anomalies earn significant alphas in the modern era, including large alphas for BAB and momentum, despite their disappearance in the baseline anomalies. In the modern era, the enhanced portfolios earn alphas ranging between 87 basis per month (10.44% annualized for IVOL) to 169 basis points (20.28% annualized) for momentum.

Panel C of Table 5 helps clarify the disappearance of BAB and momentum as baseline strategies. While the MAX-enhanced versions of these anomalies generate large positive alphas, the max weakened versions generate large negative alphas. The baseline portfolios conceal this cross-

[13]The positive pricing of idiosyncratic risk for max-weakened portfolios is consistent with the proposed model since the behavioral traders overweight both tails of the distribution, and max-weakened portfolios effectively neutralize the right tail, enabling the effect of the left-tail to become more visible. However, the positive pricing of idiosyncratic risk is not robust as the statistical significance disappears in the modern era period. Nominally, we do consistently find that the CAPM alpha for max-weakened IVOL portfolios is negative in every analysis, ranging from -17 basis points (2.04% annualized) to -36 basis points (-4.32% annualized) for max-weakened IVOL portfolios.

Table 4. Portfolio Performance: Large Cap Stocks

Panel A	(1)	(2)	(3)	(4)	(5)	(6)
Baseline	IVOL	BAB	WML	UMO	MIN	MAX
α	0.35^{**}	0.45^{***}	0.53^{***}	0.63^{***}	0.47^{**}	0.37**
	(2.24)	(2.62)	(3.01)	(5.24)	(2.43)	(2.21)
β	-0.58^{***}	-0.81^{***}	-0.11	-0.31^{***}	-0.89^{***}	-0.59***
	(-10.32)	(-11.84)	(-1.25)	(-5.27)	(-13.42)	(-9.05)
N	725	666	725	618	725	725
Adj. R-squared	0.35	0.44	0.01	0.19	0.46	0.35
Panel B						
Enhanced	IVOL	BAB	WML	UMO	MIN	
α	0.74^{***}	0.62^{**}	1.12^{***}	1.0^{***}	0.85^{***}	
	(2.95)	(2.27)	(4.85)	(3.64)	(3.08)	
β	-0.87^{***}	-1.25^{***}	-0.69^{***}	-0.81^{***}	-1.31^{***}	
	(-8.95)	(-12.82)	(-6.16)	(-8.01)	(-13.37)	
N	725	666	720	618	725	
Adj. R-squared	0.30	0.45	0.20	0.32	0.43	
Panel C						
Weakened	IVOL	BAB	WML	UMO	MIN	
α	-0.36^{**}	0.07	0.15	0.4^{*}	-0.19	
	(-2.24)	(0.45)	(0.53)	(1.95)	(-1.51)	
β	0.24^{***}	-0.03	0.55^{***}	0.42^{***}	-0.09^{**}	
	(4.09)	(-0.77)	(6.32)	(6.11)	(-2.31)	
N	725	666	720	618	725	
Adj. R-squared	0.06	0.00	0.13	0.14	0.01	
Panel D						
Enhanced - Weakened	IVOL	BAB	WML	UMO	MIN	
α	1.11^{***}	0.55^{*}	0.97^{***}	0.6	1.04^{***}	
	(3.24)	(1.85)	(2.58)	(1.50)	(3.57)	

This table reports the performance of the baseline (Panel A), enhanced (Panel B) and weakened (Panel C) portfolios in Table 2 for large cap stocks defined as stocks with a market cap above the NYSE median market cap for each month. Panel D reports the performance of the enhanced minus the weakened portfolio. *IVOL* represents idiosyncratic volatility anomaly. *BAB* represents betting against beta. *WML* represents momentum. *UMO* represents the mispricing Stambaugh and Yuan (2017). *MIN* represents the left-tail risk. *MAX* represents maximum return. α, in percent, is the intercept of regressing portfolio return on market excess return. β is the coefficient on the market excess return. The sample period is July 1963 to December 2023 (July-1968 to December-2023 for *BAB*, June-1965 to January-2017 for *UMO*). The test statistics, in parentheses, are based on Newey-West standard errors with 12 lags. ***,**, and * represent statistical significance at 1%, 5%, and 10%, respectively.

sectional heterogeneity in the anomalies between max-enhanced and max-weakened versions, giving the appearance that the anomaly has disappeared.

All of the anomaly alphas are nominally negative in Panel C, consistent with the model's intuition that max-weakened portfolios effectively neutralize the right tail and instead lead to a focus on the downside risk of the anomalies which should be compensated with a positive premium (corresponding to negative CAPM alphas). As noted by Stambaugh, Yu and Yuan (2015), underpricing is generally weaker than overpricing since buying is easier than shorting. Consistent with that intuition, our results for weakened portfolios are mostly insignificant and the evidence for underpricing (e.g., a positive premium for high idiosyncratic volatility or high left tail risk) is not robust in our data, especially compared to the large and robust effects that we find for overpricing.

Panel D of Table 5 shows that in the modern era there is a notably large gap in the performance of max-enhanced and max-weakened portfolios. Panel D shows that a portfolio that takes a long position in the max-enhanced version of an anomaly and shorts the corresponding max-weakened version earns an alpha ranging between 115 basis points (13.8% annualized) for IVOL to 259 basis points (31.08% annualized) for momentum. Accordingly, the differences in alphas between the max-enhanced and max-weakened portfolios in the modern era are all large and significant.

We next consider both limits to arbitrage jointly and investigate whether our results hold for the anomalies restricted to large cap stocks in the modern eta. The results are shown in Table 6. Panel A documents that the baseline anomaly alphas are significant except for BAB and WML.

Panel B shows that the max-enhanced IVOL, WML, UMO, and MIN anomalies all generate large and significant alphas even when restricted to large cap stocks in the modern era, further supporting Hypothesis 1. This suggests that the performance of the max-enhanced portfolios might be an equilibrium effect as predicted by the model, as it survives strong market forces that push prices toward equilibrium. Since markets for large cap stocks in the modern era are characterized by high competition, low trading costs, and high liquidity, the performance of the max-enhanced portfolios despite these forces suggests the basic CAPM anomalies have not disappeared. While the max-enhanced BAB anomaly is not significant despite its large CAPM alpha of 92 basis points per month, the strategy that takes a long position in max-enhanced BAB and shorts max-weakened BAB still earns 1.27% per month, significant at the 0.01 level, as shown in Panel D. Panel D of Table 6 further shows that all max-enhanced anomalies generate significantly larger CAPM alphas than

Table 5. Portfolio Performance in the Modern Era (Post 2005)

Panel A	(1)	(2)	(3)	(4)	(5)	(6)
Baseline	IVOL	BAB	WML	UMO	MIN	MAX
α	0.53^{**}	0.35	0.35	0.72^{***}	0.64^{**}	0.72***
	(2.41)	(0.99)	(1.14)	(3.52)	(2.07)	(3.46)
β	-0.57^{***}	-0.95^{***}	-0.45^{***}	-0.52^{***}	-1.0^{***}	-0.65***
	(-7.23)	(-13.98)	(-3.52)	(-4.62)	(-13.47)	(-8.92)
N	215	215	215	132	215	215
Adj. R-squared	0.33	0.47	0.13	0.36	0.45	0.40
Panel B						
Enhanced	IVOL	BAB	WML	UMO	MIN	
α	0.87^{**}	0.96^{**}	1.69^{***}	1.23^{***}	1.03^{**}	
	(2.37)	(2.03)	(3.87)	(2.68)	(2.02)	
β	-0.82^{***}	-1.38^{***}	-1.18^{***}	-1.0^{***}	-1.46^{***}	
	(-6.24)	(-11.97)	(-5.66)	(-8.40)	(-11.52)	
N	215	215	215	132	215	
Adj. R-squared	0.26	0.45	0.31	0.38	0.38	
Panel C						
Weakened	IVOL	BAB	WML	UMO	MIN	
α	-0.28	-0.45	-0.9^{**}	-0.41^{**}	-0.3	
	(-1.08)	(-1.27)	(-2.31)	(-1.99)	(-1.23)	
β	0.46^{***}	-0.01	0.52^{***}	0.45^{***}	-0.06	
	(7.83)	(-0.09)	(7.31)	(5.88)	(-1.49)	
N	215	215	215	132	215	
Adj. R-squared	0.23	0.00	0.14	0.20	0.00	
Panel D						
Enhanced - Weakened	IVOL	BAB	WML	UMO	MIN	
α	1.15^{**}	1.41^{***}	2.59^{***}	1.65^{***}	1.34^{**}	
	(2.31)	(3.69)	(4.85)	(3.83)	(2.57)	

This table reports the performance of the baseline (Panel A), enhanced (Panel B) and weakened (Panel C) portfolios in Table 2 in the modern era (post 2005). Panel D reports the performance of the enhanced minus the weakened portfolio. *IVOL* represents idiosyncratic volatility anomaly. *BAB* represents betting against beta. *WML* represents momentum. *UMO* represents the mispricing of Stambaugh and Yuan (2017). *MIN* represents the left-tail risk. *MAX* represents maximum return. α, in percent, is the intercept of regressing portfolio return on market excess return. β is the coefficient on the market excess return. The sample period is January 2006 to December 2023 (January-2006 to January-2017 for *UMO*). The test statistics, in parentheses, are based on Newey-West standard errors with 12 lags. ***,**, and * represent statistical significance at 1%, 5%, and 10%, respectively.

Table 6. Portfolio Performance in the Modern Era: Large Cap Stocks

Panel A	(1)	(2)	(3)	(4)	(5)	(6)
Baseline	IVOL	BAB	WML	UMO	MIN	MAX
α	0.6**	0.29	0.54	0.82***	0.74**	0.67***
	(2.43)	(0.73)	(1.58)	(3.73)	(2.33)	(3.14)
β	−0.63***	−0.98***	−0.61***	−0.54***	−1.07***	-0.68***
	(−5.56)	(−13.75)	(−4.11)	(−4.80)	(−13.45)	(-8.39)
N	215	215	215	132	215	215
Adj. R-squared	0.30	0.45	0.20	0.37	0.42	0.37

Panel B **Enhanced**	IVOL	BAB	WML	UMO	MIN
α	0.89**	0.92	1.31**	1.34***	1.38**
	(1.98)	(1.56)	(2.36)	(3.18)	(2.53)
β	−0.7***	−1.43***	−1.28***	−0.98***	−1.63***
	(−4.77)	(−9.52)	(−6.15)	(−9.11)	(−10.44)
N	215	215	215	132	215
Adj. R-squared	0.17	0.40	0.26	0.38	0.35

Panel C **Weakened**	IVOL	BAB	WML	UMO	MIN
α	−0.17	−0.35	−0.68**	−0.08	−0.27
	(−0.68)	(−0.89)	(−2.06)	(−0.24)	(−1.35)
β	0.48***	−0.06	0.44***	0.31***	−0.09
	(6.46)	(−0.56)	(6.27)	(3.27)	(−1.58)
N	215	215	215	132	215
Adj. R-squared	0.23	0.00	0.10	0.12	0.01

Panel D **Enhanced - Weakened**	IVOL	BAB	WML	UMO	MIN
α	1.06*	1.27***	1.99***	1.42**	1.66***
	(1.85)	(2.93)	(3.27)	(2.55)	(2.86)

This table reports the performance of the baseline (Panel A), enhanced (Panel B) and weakened (Panel C) portfolios in Table 2 in the modern era (post 2005) for large cap stocks defined as stocks with a market cap above the NYSE median market cap for each month. Panel D reports the performance of the enhanced minus the weakened portfolio. *IVOL* represents idiosyncratic volatility anomaly. *BAB* represents betting against beta. *WML* represents momentum. *UMO* represents the mispricing of Stambaugh and Yuan (2017). *MIN* represents the left-tail risk. *MAX* represents maximum return. α, in percent, is the intercept of regressing portfolio return on market excess return. β is the coefficient on the market excess return. The sample period is January 2006 to December 2023 (January-2006 to January-2017 for *UMO*). The test statistics, in parentheses, are based on Newey-West standard errors with 12 lags. ***,**, and * represent statistical significance at 1%, 5%, and 10%, respectively.

the corresponding max-weakened portfolios, providing further support for hypothesis 3. As before, the CAPM alphas for the enhanced-minus-weakened strategies is economically large, ranging from 1.06% per month for IVOL to 1.99% percent per month for momentum.

6 Time Variation in Anomalies due to Lottery Demand

We have thus far focused on the model prediction that the CAPM alphas are increasing in the difference in maximum returns between the short and long legs of the anomalies. when this difference is sufficiently large such that the CAPM alphas are positive, as one would expect for the max-enhanced portfolios, then the model predicts that the alphas are amplified by the proportion of lottery demand traders in the market. We now turn to this prediction which is the subject of our Hypothesis 4.

To measure time-variation in lottery demand, we use the measure of speculative impact from Ghazi et al. (2024). That paper shows theoretically that the marginal value from participating in the stock market for a representative lottery demand trader (modeled as an investor with NEO-EU probability weighting) is increasing in the relative weight on the market's perceived maximum return (the equivalent of our θ parameter) and decreasing in market volatility. That is, lottery demand is predicted to be stronger in markets with high optimism toward lottery-like returns during calm periods. They use their theoretical analysis to motivate the measure, $-\hat{q}(1-\hat{\theta})$, as a measure of aggregate lottery demand, where $\hat{\theta}$ is the measure of market optimism (the weight on the perceived market maximum return) from Ghazi, Schneider and Strauss (2024) estimated from a model with a representative NEO-EU investor, and $\hat{q}$ is their series of market volatility estimated from a GARCH model. Note that this measure which they refer to as speculative impact is increasing in $\hat{\theta}$, and it is decreasing in $\hat{q}$. Ghazi et al. (2024) show using predictive regressions that the measure predicts time variation in the raw returns of the MAX, IVOL, WML, and UMO anomalies. In this section, we apply the speculative impact measure as a theory-based proxy for aggregate lottery demand to test if it predicts time variation in the *CAPM alphas* of these baseline anomalies as well as for the alphas of the max-enhanced and max-weakened versions and for those of the BAB and MIN anomalies.

The results are presented in Table 7. Panel A of the table shows that the measure of aggregate lottery demand positively predicts the CAPM residuals (i.e., the CAPM alphas) for each of the six baseline anomalies. Panel B shows that the measure positively predicts each of the enhanced anomalies. These findings support the predictions of the theory (and Hypothesis 4) that the anomalies are amplified by lottery demand (providing the CAPM alphas are initially positive). Since the max-weakened versions do not generally have CAPM alphas that are significantly greater than zero, we do not expect lottery demand to amplify those anomalies, and generally the measure does not predict the weakened anomalies, except for UMO (with a lower t-statistic compared to the enhanced version). In our robustness analyses, we show that the results in Table 7 are not subsumed by market sentiment (the Baker and Wurgler (2006) sentiment index) or by market volatility (the measure $\hat{q}$ by itself).

7 Additional Analyses

In this section, we perform three sets of additional analyses: (1) We investigate if our results hold using an alternative method for computing *Max* (analogous to how we computed *Min*, based on value-at-risk (Atilgan et al., 2020) but applied to the right tail); (2) We investigate if our results for enhanced and weakened portfolios hold when using the full CRSP universe of stocks (including microcaps); (3) We investigate if our results for time variation in lottery demand are subsumed by market sentiment or market volatility. In each case, we find that our results are robust and in line with the predictions of the model.

7.1 The MAX Effect as an Explanation for the IVOL Puzzle

Although the MAX effect has been used as an explanation for the IVOL puzzle (Bali, Cakici and Whitelaw, 2011), Hou and Loh (2016) argues that the MAX effect does not provide a valid empirical explanation for the IVOL puzzle. The key issue as noted by Hou and Loh (2016) is that *Max* and *Ivol* have a high correlation in the cross-section which they noted was near 0.90 and which is 0.86 in our data as shown in Table 1. Hou and Loh (2016) argues that this very high correlation indicates that *Max* is just a proxy for *Ivol*. The MAX effect still provides a conceptual explanation for IVOL (since *Max* stocks are plausibly overvalued due to lottery demand, and high *Ivol* stocks naturally have higher maximum returns than low *Ivol* stocks). However, the high collinearity between *Max*

Table 7. Predicting Portfolio Performance with Lottery Demand

Panel A	(1)	(2)	(3)	(4)	(5)	(6)
Baseline	$IVOL_{t+1}$	BAB_{t+1}	WML_{t+1}	UMO_{t+1}	MIN_{t+1}	MAX_{t+1}
$-\hat{q}_t(1-\hat{\theta}_t)$	0.74^{***}	0.67^{*}	1.18^{**}	0.49^{***}	0.62^{**}	0.66^{***}
	(2.68)	(1.73)	(2.23)	(2.82)	(2.14)	(3.27)
N	198	198	198	126	198	198
Adj. R-squared	0.05	0.02	0.06	0.04	0.02	0.04
Panel B						
Enhanced	$IVOL_{t+1}$	BAB_{t+1}	WML_{t+1}	UMO_{t+1}	MIN_{t+1}	
$-\hat{q}_t(1-\hat{\theta}_t)$	1.14^{***}	0.88^{*}	1.75^{***}	1.27^{***}	1.52^{***}	
	(2.83)	(1.86)	(2.65)	(3.58)	(3.21)	
N	198	198	198	126	198	
Adj. R-squared	0.04	0.02	0.06	0.08	0.04	
Panel C						
Weakened	$IVOL_{t+1}$	BAB_{t+1}	WML_{t+1}	UMO_{t+1}	MIN_{t+1}	
$-\hat{q}_t(1-\hat{\theta}_t)$	-0.08	0.29	0.35	0.46^{**}	0.13	
	(-0.37)	(1.01)	(1.09)	(2.52)	(0.51)	
N	198	198	198	126	198	
Adj. R-squared	-0.005	0.000	0.000	0.017	-0.003	

This table reports the performance predictability of baseline (Panel A), enhanced (Panel B), and weakened (Panel C) portfolios in Table 2 with a measure of aggregate lottery demand ($-\hat{q}(1-\hat{\theta})$) from Ghazi et al. (2024). First, portfolio returns are regressed on market excess return. The residuals of this regression are regressed on the lagged lottery demand. The second estimated regression is reported. *IVOL* represents idiosyncratic volatility anomaly. *BAB* represents betting against beta. *WML* represents momentum. *UMO* represents the mispricing of Stambaugh and Yuan (2017). *MIN* represents the left-tail risk. MAX represent maximum return. The sample period is July 2006 to December 2022 (July-2006 to January-2017 for *UMO*). The test statistics, in parentheses, are based on Newey-West standard errors with 12 lags. ***,**, and * represent statistical significance at 1%, 5%, and 10%, respectively.

and *Ivol* makes this prediction difficult to test.

In this section, we introduce an alternative measure for *Max* that is analogous to how Atilgan et al. (2020) compute *Min* using value-at-risk, except that we apply the method to the right tail. Under this alternative measure, *MAax5*, the perceived maximum return is calculated as the highest 5% of return observations over the past year.[14] The *Max5* measure has a substantially lower correlation with *Ivol* of 0.64 as shown in Table 1, revealing that the two series are clearly not identical. Since the theory in Section 2 includes an important role for *Max* but does not specify how *Max* should be calculated empirically, either the *Max* measure in our main analyses based on Bali, Cakici and Whitelaw (2011) or the *Max* measure in this section, based on Atilgan et al. (2020) are valid measures that can be used to test the theoretical predictions. The natural question that arises is whether *Max5* also works in explaining the CAPM anomalies. We find that it does.

Table 8 presents the performance of max-enhanced and max-weakened portfolios constructed using the *Max5* measure instead of *Max*. Panel A shows that all max-enhanced portfolios earn positive and significant alphas ranging from 0.65% (7.8% annualized) for BAB to 1.35% (16.2% annualized) for momentum, supporting Hypothesis 1. Panel B shows that none of the max-weakened portfolios earn significant alphas which range from -0.25% per month for IVOL to 0.14% per month for UMO, supporting Hypothesis 2. Panel C shows that each of the max-enhanced portfolios earns a significantly higher alpha than the corresponding max-weakened portfolio, supporting Hypothesis 3. As the the max-enhanced IVOL earns a significant annualized alpha of 11.04%, while the max-weakened IVOL earns an insignificant alpha, we find that the *Max5* measure helps explain how to strengthen or eliminate the IVOL puzzle. This observation holds even though *Max5* is substantially less correlated with *Ivol* than *Max*.

7.2 Performance using All CRSP Stocks

Hou, Xue and Zhang (2020) show that many asset pricing anomalies disappear after excluding microcap stocks (those in the bottom 20% of NYSE market capitalization). To ensure that our results are not driven by microcap stocks, all of our analyses thus far have excluded microcap stocks. Since microcap stocks do comprise a substantial portion of the total number of firms in

[14] In an unreported analysis, we find very similar results for both *Max* and *Min* if they are computed as the highest 1% of return observations instead of the highest 5%.

Table 8. Enhanced and Weakened Portfolios Using an Alternative Measure of Maximum Return

Panel A	(1)	(2)	(3)	(4)	(5)
Enhanced	IVOL	BAB	WML	UMO	MIN
α	0.92***	0.65**	1.35***	1.33***	1.05***
	(3.23)	(2.22)	(4.94)	(4.52)	(3.51)
β	−1.1***	−1.32***	−1.04***	−1.06***	−1.42***
	(−10.79)	(−12.79)	(−10.04)	(−10.79)	(−14.00)
N	725	666	720	618	725
Adj. R-squared	0.35	0.46	0.30	0.42	0.43
Panel B					
Weakened	IVOL	BAB	WML	UMO	MIN
α	−0.25	−0.11	0.13	0.14	0.1
	(−1.18)	(−0.49)	(0.46)	(0.54)	(0.56)
β	0.71***	0.38***	0.81***	0.8***	0.44***
	(10.14)	(8.00)	(10.53)	(10.05)	(8.17)
N	725	666	724	618	725
Adj. R-squared	0.27	0.12	0.23	0.29	0.17
Panel C					
Enhanced - Weakened	IVOL	BAB	WML	UMO	MIN
α	1.18***	0.75*	1.24***	1.19**	0.96**
	(2.68)	(1.94)	(2.82)	(2.40)	(2.30)

This table reports the performance of the enhanced (Panel A) and weakened (Panel B) portfolios in Table 2 using an alternative measure of maximum return defined as the 95^{th} percentile of daily returns over the past year. Panel C reports the performance of the enhanced minus the weakened portfolio. *IVOL* represents idiosyncratic volatility anomaly. *BAB* represents betting against beta. *WML* represents momentum. *UMO* represents the mispricing of Stambaugh and Yuan (2017). *MIN* represents the left-tail risk. *MAX* represents maximum return. α, in percent, is the intercept of regressing portfolio return on market excess return. β is the coefficient on the market excess return. The sample period is July 1963 to December 2023 (July-1968 to December-2023 for *BAB*, June-1965 to January-2017 for *UMO*). The test statistics, in parentheses, are based on Newey-West standard errors with 12 lags. ***,**, and * represent statistical significance at 1%, 5%, and 10%, respectively.

the CRSP universe, we investigate in this section whether our results continue to hold if all CRSP stocks on the NYSE, NASDAQ, and AMEX exchanges (including microcaps) are included.

The results are shown in Table 9. Panel A confirms that the baseline anomalies exist when including all stocks as each of the CAPM alphas are significant. Panel B shows that the alphas of all max-enhanced portfolios are significant, supporting Hypothesis 1. Panel C shows that none of the max-weakened alphas are significant at the 0.05 level (with only the UMO alpha being significant at the 0.10 level), supporting Hypothesis 2. Panel D shows that each of the max-enhanced alphas is significantly larger than the corresponding max-weakened alpha, supporting Hypothesis 3.

7.3 Time Variation in Lottery Demand versus Sentiment

We next investigate whether the lottery demand results from Section 6 are explained by market sentiment. We run the same regressions as in Table 7 except that we now control for the Baker and Wurgler (2006) market sentiment index. The results are shown in Table 10. The table reveals that our measure for aggregate lottery demand continues to predict all six baseline anomalies (in Panel A) and all enhanced portfolios (in Panel B) in the presence of sentiment. In contrast, sentiment is only significant for BAB anomaly, and any predictive power of sentiment for the other anomalies is subsumed by the measure for aggregate lottery demand.

In one further robustness check, we investigate whether the lottery demand results from Section 6 can be explained by market volatility, $\hat{q}$, alone. We run analogous regressions controlling for $\hat{q}$. The results are shown in Table 11. In the presence of $\hat{q}$, which is also a component of our measure for aggregate lottery demand, the measure for aggregate lottery demand remains significant for five of the six baseline anomalies (all except forBAB), as shown in Panel A. In Panel B, we find that the measure for aggregate lottery demand remains significant in predicting the enhanced versions of all anomalies except for BAB. Although our measure does not predict BAB when controlling for $\hat{q}$, the BAB coefficient for $\hat{q}$ is also not significant. In contrast, the coefficient on $\hat{q}$ is only significant for UMO. We conclude that the performance of the measure of aggregate lottery demand is not subsumed by market sentiment or market volatility.

Table 9. Portfolio Performance: All Stocks

Panel A	(1)	(2)	(3)	(4)	(5)	(6)
Baseline	IVOL	BAB	WML	UMO	MIN	MAX
α	0.54^{***}	0.42^{**}	0.8^{***}	0.76^{***}	0.59^{***}	0.46^{**}
	(2.87)	(2.18)	(4.18)	(6.12)	(2.65)	(2.53)
β	-0.67^{***}	-0.86^{***}	-0.26^{**}	-0.34^{***}	-0.97^{***}	-0.67^{***}
	(-9.88)	(-12.31)	(-2.56)	(-5.87)	(-12.65)	(-9.60)
N	725	666	725	618	725	725
Adj. R-squared	0.32	0.45	0.04	0.22	0.41	0.35
Panel B						
Enhanced	IVOL	BAB	WML	UMO	MIN	
α	1.11^{***}	0.8^{***}	1.69^{***}	1.47^{***}	1.29^{***}	
	(3.77)	(2.61)	(5.74)	(5.16)	(3.71)	
β	-0.85^{***}	-1.32^{***}	-0.9^{***}	-0.9^{***}	-1.39^{***}	
	(-7.63)	(-12.57)	(-6.98)	(-9.34)	(-10.96)	
N	725	666	725	618	725	
Adj. R-squared	0.23	0.43	0.20	0.36	0.35	
Panel C						
Weakened	IVOL	BAB	WML	UMO	MIN	
α	-0.2	-0.01	0.08	0.34^{*}	-0.09	
	(-1.30)	(-0.06)	(0.29)	(1.75)	(-0.73)	
β	0.35^{***}	-0.05	0.59^{***}	0.41^{***}	-0.07^{**}	
	(5.77)	(-1.04)	(6.81)	(5.75)	(-1.97)	
N	725	666	725	618	725	
Adj. R-squared	0.13	0.00	0.15	0.13	0.01	
Panel D						
Enhanced - Weakened	IVOL	BAB	WML	UMO	MIN	
α	1.31^{***}	0.81^{**}	1.62^{***}	1.13^{***}	1.38^{***}	
	(3.40)	(2.52)	(3.67)	(2.72)	(3.76)	

This table reports the performance of the baseline (Panel A), enhanced (Panel B) and weakened (Panel C) portfolios in Table 2 for all stocks without excluding micro caps. Panel D reports the performance of the enhanced minus the weakened portfolio. *IVOL* represents idiosyncratic volatility anomaly. *BAB* represents betting against beta. *WML* represents momentum. *UMO* represents the mispricing of Stambaugh and Yuan (2017). *MIN* represents the left-tail risk. *MAX* represents maximum return. α, in percent, is the intercept of regressing portfolio return on market excess return. β is the coefficient on the market excess return. The sample period is July 1963 to December 2023 (July-1968 to December-2023 for *BAB*, June-1965 to January-2017 for *UMO*). The test statistics, in parentheses, are based on Newey-West standard errors with 12 lags. ***,**, and * represent statistical significance at 1%, 5%, and 10%, respectively.

Table 10. Predicting Portfolio Performance with Lottery Demand: Controlling for Sentiment

Panel A	(1)	(2)	(3)	(4)	(5)	(6)
Baseline	$IVOL_{t+1}$	BAB_{t+1}	WML_{t+1}	UMO_{t+1}	MIN_{t+1}	MAX_{t+1}
$-\hat{q}_t(1-\hat{\theta}_t)$	0.65^{**}	0.76^{*}	1.38^{**}	0.7^{**}	0.55^{*}	0.61^{***}
	(2.12)	(1.86)	(2.49)	(2.00)	(1.65)	(2.74)
BW_t	0.0	−0.0	-0.01^{**}	−0.01	0.0	0.0
	(0.97)	(−1.07)	(−2.00)	(−0.85)	(0.69)	(0.71)
N	198	198	198	126	198	198
Adj. R-squared	0.05	0.02	0.06	0.04	0.01	0.03
Panel B						
Enhanced	$IVOL_{t+1}$	BAB_{t+1}	WML_{t+1}	UMO_{t+1}	MIN_{t+1}	
$-\hat{q}_t(1-\hat{\theta}_t)$	0.91^{**}	1.16^{**}	1.87^{***}	1.82^{***}	1.23^{**}	
	(2.04)	(2.32)	(2.66)	(3.18)	(2.26)	
BW_t	0.01	-0.01^{***}	−0.01	−0.03	0.01	
	(1.56)	(−2.59)	(−1.09)	(−1.21)	(1.45)	
N	198	198	198	126	198	
Adj. R-squared	0.04	0.02	0.06	0.09	0.04	
Panel C						
Weakened	$IVOL_{t+1}$	BAB_{t+1}	WML_{t+1}	UMO_{t+1}	MIN_{t+1}	
$-\hat{q}_t(1-\hat{\theta}_t)$	−0.19	0.47	0.55^{*}	0.6^{*}	0.16	
	(−0.78)	(1.44)	(1.71)	(1.65)	(0.59)	
BW_t	0.01	-0.01^{**}	-0.01^{*}	−0.01	−0.0	
	(1.21)	(−2.23)	(−1.70)	(−0.62)	(−0.38)	
N	198	198	198	126	198	
Adj. R-squared	−0.004	0.006	0.002	0.011	−0.008	

This table reports the performance predictability of baseline (Panel A), enhanced (Panel B), and weakened (Panel C) portfolios in Table 2 with a measure of aggregate lottery demand ($-\hat{q}(1-\hat{\theta})$, from (Ghazi et al., 2024)) controlling for the market sentiment First, portfolio returns are regressed on market excess return. The residuals of this regression are regressed on the lagged lottery demand and the lagged sentiment measure of Baker and Wurgler (2006), BW. The second estimated regression is reported. *IVOL* represents idiosyncratic volatility anomaly. *BAB* represents betting against beta. *WML* represents momentum. *UMO* represents the mispricing of Stambaugh and Yuan (2017). *MIN* represents the left-tail risk. *MAX* represents maximum return. The sample period is July 2006 to December 2022 (July-2006 to January-2017 for *UMO*). The test statistics, in parentheses, are based on Newey-West standard errors with 12 lags. ***,**, and * represent statistical significance at 1%, 5%, and 10%, respectively.

Table 11. Predicting Performance with Lottery Demand: Controlling for Market Volatility

Panel A	(1)	(2)	(3)	(4)	(5)	(6)
Baseline	$IVOL_{t+1}$	BAB_{t+1}	WML_{t+1}	UMO_{t+1}	MIN_{t+1}	MAX_{t+1}
$-\hat{q}_t(1-\hat{\theta}_t)$	1.0^{**}	0.21	1.41^{*}	1.4^{***}	1.02^{**}	0.92^{**}
	(2.07)	(0.48)	(1.92)	(3.91)	(2.16)	(2.29)
$\hat{q}_t$	0.21	-0.36	0.18	0.77^{***}	0.31	0.2
	(0.96)	(-1.06)	(0.51)	(3.70)	(1.26)	(0.91)
N	198	198	198	126	198	198
Adj. R-squared	0.05	0.02	0.06	0.08	0.01	0.04
Panel B						
Enhanced	$IVOL_{t+1}$	BAB_{t+1}	WML_{t+1}	UMO_{t+1}	MIN_{t+1}	
$-\hat{q}_t(1-\hat{\theta}_t)$	1.79^{**}	0.5	2.8^{***}	2.24^{***}	2.0^{**}	
	(2.44)	(0.66)	(2.76)	(4.17)	(2.47)	
$\hat{q}_t$	0.51	-0.29	0.83	0.82^{**}	0.38	
	(1.41)	(-0.75)	(1.64)	(2.07)	(0.74)	
N	198	198	198	126	198	
Adj. R-squared	0.04	0.01	0.06	0.09	0.03	
Panel C						
Weakened	$IVOL_{t+1}$	BAB_{t+1}	WML_{t+1}	UMO_{t+1}	MIN_{t+1}	
$-\hat{q}_t(1-\hat{\theta}_t)$	0.13	-0.9	-0.19	0.53	0.38	
	(0.29)	(-1.20)	(-0.34)	(1.59)	(0.79)	
$\hat{q}_t$	0.17	-0.94	-0.42	0.06	0.2	
	(0.49)	(-1.53)	(-0.89)	(0.22)	(0.68)	
N	198	198	198	126	198	
Adj. R-squared	-0.008	0.032	-0.001	0.009	-0.006	

This table reports the performance predictability of baseline (Panel A), enhanced (Panel B), and weakened (Panel C) portfolios in Table 2 with a measure of aggregate lottery demand ($-\hat{q}(1-\hat{\theta})$, from (Ghazi et al., 2024)) controlling for market volatility. First, portfolio returns are regressed on market excess return. The residuals of this regression are regressed on the lagged lottery demand and the lagged market volatility form a GARCH model. The second estimated regression is reported. *IVOL* represents idiosyncratic volatility anomaly. *BAB* represents betting against beta. *WML* represents momentum. *UMO* represents the mispricing of Stambaugh and Yuan (2017). *MIN* represents the left-tail risk. *MAX* represent maximum return. The sample period is July 2006 to December 2022 (July-2006 to January-2017 for *UMO*). The test statistics, in parentheses, are based on Newey-West standard errors with 12 lags. ***,**, and * represent statistical significance at 1%, 5%, and 10%, respectively.

8 Conclusion

In this paper, we have shown that extending the CAPM through probability weighting function leads to a tractable and transparent representation of asset prices and the CAPM alpha. The specific prediction that deviations from the CAPM are strengthened by max-enhanced portfolios and weakened by max-weakened portfolios provides a new theoretical explanation for major CAPM anomalies, including anomalies that prior theoretical work has had difficulty explaining using very different approaches. For instance, the only prior study to directly investigate max-enhanced and max-weakened portfolios is Jacobs, Regele and Weber (2015) who did so with regard to momentum. They found that a max- weakened (max-enhanced) momentum strategy has lower (higher) momentum returns. Jacobs, Regele and Weber (2015) conclude that their findings "appear to provide a challenge for popular theories of momentum, which are based on investor overreaction (Daniel, Hirshleifer and Subrahmanyam, 1998), investor underreaction followed by overreaction (Barberis, Shleifer and Vishny, 1998; Hong and Stein, 1999), agency issues in delegated fund management (Vayanos and Woolley, 2013), credit risk (Avramov et al., 2007) or the disposition effect (Grinblatt and Han, 2005)." They remark that their findings "do not fit neatly within a specific prominent theory of momentum." However, their findings provide support for the momentum premium under the MAX-CAPM. More broadly, existing theoretical models traditionally emphasize the left-tail (as in models based on disaster risk, ambiguity aversion, or disappointment aversion), and so are unable to explain the anomalies studied here which are driven by agents overweighting the right tail.

Probability weighting models have shown promise in explaining financial market behavior at the level of individual investors and at the level of the aggregate market, with recent work turning to its implications for the cross-section. Our analysis establishes the existence and uniqueness of the equilibrium in a market with heterogeneous traders including some textbook mean-variance arbitrageurs and some lottery demand traders with a textbook probability weighting function. The model provides a microfoundation and unifying role for the MAX effect in explaining central CAPM anomalies and may serve to help bridge the gap between rational and behavioral theories of asset prices.

Appendix. Proofs

Proof of Proposition 1:

First, we start with the existence and uniqueness of the equilibrium. The mean-variance investors solve the problem in (2), which can be written as

$$\max_y y'\big(E_t P_{t+1} - R_f P_t\big) - \frac{\rho}{2} y'\Sigma\, y.$$

Using vector differentiation rules $\frac{\partial}{\partial y}(y'e) = e$, and $\frac{\partial}{\partial y}(y'\Sigma y) = 2\Sigma y$, the first-order condition of the above maximization yields

$$y^*(P_t) = \rho^{-1}\Sigma^{-1}\big(E_t P_{t+1} - R_f P_t\big). \tag{7}$$

Note that $y^*(P_t)$ is the unique maximizer of the problem in (2) since Σ is positive definite, and hence, the target function is strictly concave in y. Moreover, since Σ^{-1} is also positive definite, $y^*(P_t)$ is decreasing in P_t, and hence, it is a proper (asset) demand function of the mean-variance agents.[15]

Similarly, the optimization problem of the noise traders in (1) can be written as

$$\max_x x'\left((1-\gamma)E_t P_{t+1} + \gamma\big(\theta \overline{P}_{t+1} + (1-\theta)\underline{P}_{t+1}\big) - R_f P_t\right) - \frac{\rho}{2} x'\Sigma\, x,$$

which is strictly concave in x, and its first-order condition yields the following unique maximizer, i.e., noise traders' asset demand:

$$x^*(P_t) = \rho^{-1}\Sigma^{-1}\left((1-\gamma)E_t P_{t+1} + \gamma\big(\theta \overline{P}_{t+1} + (1-\theta)\underline{P}_{t+1}\big) - R_f P_t\right). \tag{8}$$

Finally, using the market clearing condition, $\varphi x^*(P_t) + (1-\varphi)y^*(P_t) = \mathbf{1}_n$, we have:

$$(1-\varphi)E_t P_{t+1} + \varphi\left((1-\gamma)E_t P_{t+1} + \gamma\big(\theta \overline{P}_{t+1} + (1-\theta)\underline{P}_{t+1}\big)\right) - P_t R_f = \rho\Sigma\mathbf{1}_n,$$

[15] To see that Σ^{-1} is positive definite, i.e., $z'\Sigma^{-1}z > 0$ for any n-vector z, define $w \coloneqq \Sigma^{-1}z$, which exists because Σ is positive definite and hence, invertible. Thus, $z = \Sigma w$, and $z'\Sigma^{-1}z = w'\Sigma'\Sigma^{-1}\Sigma w = w'\Sigma w > 0$, where we also used the fact a the real positive definite matrix is symmetric.

which can be immediately solved for P_t as follows:

$$P_t = R_f^{-1}\Big((1-\varphi\gamma)E_tP_{t+1} + \varphi\gamma\big(\theta\overline{P}_{t+1} + (1-\theta)\underline{P}_{t+1}\big) - \rho\Sigma\mathbf{1}_n\Big). \tag{9}$$

Note that the right-hand side only contains the states variables, and P_t is the unique vector that solves the market-clearing condition. However, we still need to make sure that $P_t \gg 0$, i.e., P_t must be (element-wise) strictly positive. Specifically, $P_t \gg 0$ is achieved if

$$\rho\mathbf{1}_n \ll \Sigma^{-1}\Big((1-\varphi\gamma)E_tP_{t+1} + \varphi\gamma\big(\theta\overline{P}_{t+1} + (1-\theta)\underline{P}_{t+1}\big)\Big). \tag{10}$$

Thus, ρ must be smaller than the minimum of the elements of the vector on the right-hand side of (10), which completes the proof of existence and uniqueness of the equilibrium.

Next, we derive the expression of excess returns. The expressions of x^* and y^* as solutions to investor problems in (1) and (2) are derived in (7) and (8), respectively. For later calculations in this proof, it is useful to rewrite x^* as the following:

$$x^* = \rho^{-1}\Sigma^{-1}\Big((1-\gamma)E_tP_{t+1} + A_tP_t - P_tR_f\Big),$$

where A_t is the $n \times n$ diagonal matrix with the (j,j) element $a_{j,t} := \gamma\big(\theta\overline{R}_{j,t+1} + (1-\theta)\underline{R}_{j,t+1}\big)$. As before, substituting x^* and y^* in the market clearing condition, $\varphi x^* + (1-\varphi)y^* = \mathbf{1}_n$, yields:

$$\varphi A_tP_t + (1-\varphi\gamma)E_tP_{t+1} - P_tR_f = \rho\Sigma\mathbf{1}_n,$$

which we can rearrange to express the equilibrium price as follows.

$$\big(R_fI - \varphi A_t\big)P_t = (1-\varphi\gamma)E_tP_{t+1} - \rho\Sigma\mathbf{1}_n,$$

where I is the $n \times n$ identity matrix. Writing the above in terms of the j'th stock and dividing through by $p_{j,t}$, we have

$$(1-\varphi\gamma)E_t(R_{j,t+1}) - R_f = -\varphi a_{j,t} + \rho\frac{1}{p_{j,t}}e_j'\Sigma\mathbf{1}_n, \tag{11}$$

where e_j is the unit vector with a one in position j and zeros elsewhere, and e_j' selects the j'th row when left-multiplied. Replacing Σ in $\frac{1}{p_{j,t}} e_j' \Sigma \mathbf{1}_n$ with its definition, we have

$$\begin{aligned}
\frac{1}{p_{j,t}} e_j' \Sigma \mathbf{1}_n &= \frac{1}{p_{j,t}} e_j' E_t\big((P_{t+1})(P_{t+1} - E_t P_{t+1})'\big)\mathbf{1}_n \\
&= E_t\Big(\frac{p_{j,t+1}}{p_{j,t}}(P_{t+1} - E_t P_{t+1})'\Big)\mathbf{1}_n \\
&= Cov_t\big(R_{j,t+1}, P_{t+1}'\mathbf{1}_n\big).
\end{aligned}$$

Next, note that for the market return, we have $R_{M,t+1} = \frac{\sum_{j=1}^n p_{j,t+1}}{\sum_{i=1}^n p_{i,t}} = \sum_{j=1}^n R_{j,t+1} \frac{p_{j,t}}{\sum_{i=1}^n p_{i,t}}$, and thus, $R_{M,t+1} = \sum_{j=1}^n w_{j,t} R_{j,t+1}$, with weights $w_{j,t} \coloneqq \frac{p_{j,t}}{\sum_{i=1}^n p_{i,t}}$. Combining this with the above expression for $\frac{1}{p_{j,t}} e_j' \Sigma \mathbf{1}_n$, we have

$$\begin{aligned}
\frac{1}{p_{j,t}} e_j' \Sigma \mathbf{1}_n &= Cov_t\big(R_{j,t+1}, P_{t+1}'\mathbf{1}_n\big) \\
&= Cov_t\big(R_{j,t+1}, \sum_{j=1}^n \frac{p_{j,t+1}}{p_{j,t}} p_{j,t}\big) \\
&= Cov_t\big(R_{j,t+1}, \sum_{j=1}^n R_{j,t+1} w_{j,t}\big) \sum_{i=1}^n p_{i,t},
\end{aligned}$$

and thus, we have $\frac{1}{p_{j,t}} e_j' \Sigma \mathbf{1}_n = Cov_t\big(R_{j,t+1}, R_{M,t+1}\big) P_t' \mathbf{1}_n$, and we can write (11) as the following

$$(1 - \varphi\gamma) E_t(R_{j,t+1}) - R_f = -\varphi a_{j,t} + \rho\, Cov_t(R_{j,t+1}, R_{M,t+1}) P_t' \mathbf{1}_n. \tag{12}$$

Multiplying the above equation by $w_{j,t}$ and summing over j we find

$$(1 - \varphi\gamma) E_t(R_{M,t+1}) - R_f = -\varphi\, a_{M,t} + \rho\, Var_t(R_{M,t+1}) P_t' \mathbf{1}_n,$$

where $a_{M,t} \coloneqq \gamma\big(\theta \overline{R}_{M,t+1} + (1-\theta) \underline{R}_{M,t+1}\big)$. Rearranging terms, we find the value of the market

$$P_t' \mathbf{1}_n = \frac{(1 - \varphi\gamma) E_t(R_{M,t+1}) - R_f + \varphi\, a_{M,t}}{\rho Var_t(R_{M,t+1})}.$$

Substituting $P_t'\mathbf{1}_n$ into (12), we find the expected return formula for asset j

$$(1-\varphi\gamma)E_t(R_{j,t+1}) - R_f = -\varphi a_{j,t} + \rho\, Cov_t(R_{j,t+1}, R_{M,t+1})\frac{(1-\varphi\gamma)E_t(R_{M,t+1}) - R_f + \varphi\, a_{M,t}}{\rho Var_t(R_{M,t+1})}$$
$$(1-\varphi\gamma)E_t(R_{j,t+1}) - R_f = -\varphi a_{j,t} + \beta_j\Big((1-\varphi\gamma)E_t(R_{M,t+1}) - R_f + \varphi\, a_{M,t}\Big).$$

Dividing through by $(1-\varphi\gamma)$ and rearranging terms, we find

$$E_t(R_{j,t+1}) - R_f = \Big(\frac{\varphi}{1-\varphi\gamma}\Big)\big(-a_{j,t} + \beta_j a_{M,t} + \gamma(1-\beta_j)R_f\big) + \beta_j\big(E_t(R_{M,t+1}) - R_f\big),$$

which we can write as a generalization of the standard CAPM,

$$E_t(R_{j,t+1}) - R_f = \alpha_j + \beta_j\big(E_t(R_{M,t+1}) - R_f\big),$$

with the CAPM α being

$$\alpha_j = \left(\frac{\varphi\gamma}{1-\varphi\gamma}\right)\Big(\theta(\beta_j\overline{R}_{M,t+1} - \overline{R}_{j,t+1}) + (1-\theta)(\beta_j\underline{R}_{M,t+1} - \underline{R}_{j,t+1}) + (1-\beta_j)R_f\Big).$$

This concludes the proof of the proposition. □

Proof of Corollary 2:

Part (1). Using the expression of the equilibrium price in (9), we have

$$R_f P_t = (1-\varphi\gamma)E_t P_{t+1} + \varphi\gamma\big(\theta\overline{P}_{t+1} + (1-\theta)\underline{P}_{t+1}\big) - \rho\Sigma_t\mathbf{1}_n,$$

Replacing $R_f P$ in the expressions of y^* and x^* in (7) and (8) gives us

$$x^* = \mathbf{1}_n + \gamma(1-\varphi)\rho^{-1}\Sigma_t^{-1}\Big(-E_t P_{t+1} + \big(\theta\overline{P}_{t+1} + (1-\theta)\underline{P}_{t+1}\big)\Big), \tag{13}$$
$$y^* = \mathbf{1}_n + \gamma\varphi\rho^{-1}\Sigma_t^{-1}\Big(E_t P_{t+1} - \big(\theta\overline{P}_{t+1} + (1-\theta)\underline{P}_{t+1}\big)\Big). \tag{14}$$

Given the assumption of range-based expectations and given that the maximum and minimum prices have not changed, an increase in the asset's price must be due to an increase in the expected price (e.g., arising from good news about the asset's fundamentals). Thus, from the expression of

$R_f P_t$, good news about fundamentals that increase $E_t P_{t+1}$ and keeps everything else unchanged, increases P_t and pushes the price toward the max price $\overline{P}_{t+1}$. In addition, from the expressions in (13) and (14), x^* is decreasing in $E_t P_{t+1}$ and y^* is increasing $E_t P_{t+1}$ (since the matrix Σ_t is positive definite and the rest of the scalar coefficients are positive). Thus, the same news lowers the holdings of the noise traders and raises the holdings of arbitrageurs.

Part (2). Consider an increase in the expected price $E_t\, p_{j,t+1}$, which increases $p_{j,t}$ and pushes it to toward $\overline{p}_{j,t+1}$. Moreover, it causes arbitrageurs (noise traders) to buy (sell) asset j. Hence, it only remains to show that α_j increases in $E_t\, p_{j,t+1}$. Given that this change has negligible effect on the market (i.e., individual stock weights are relatively small), to find the effect on α_j we can use Equation (5) which is reproduced below for convenience.

$$\alpha_j \coloneqq \left(\frac{\varphi\gamma}{1-\varphi\gamma}\right)\left(\theta(\beta_j \overline{R}_M - \overline{R}_j) + (1-\theta)(\beta_j \underline{R}_M - \underline{R}_j) + (1-\beta_j)R_f\right).$$

Thus, α_j is affected only through a decline in both extreme returns $\overline{R}_j$ and $\underline{R}_j$ (since $p_{j,t}$ increases), both of which increase α_j. $\square$

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